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SCHOOL SCIENCE AND MATHEMATICS

FOUNDED BY C. E. LINEBARGER

A Journal for all SCIENCE AND MATHEMATICS TEACHERS

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SCHOOL SCIENCE AND MATHEMATICS

VOL. XXXII NO. 3 MARCH, 1932 WHOLE NO. 275

DR. E. LAURENCE PALMER, NATURE STUDY EDITOR

Who does not enjoy a day in the hills or woods? Whether it be Winter or Summer, there are always so many interesting things to be seen and so much to be enjoyed in the out-of-doors. If one does not enjoy a nature ramble, if he cannot see the things of interest all around him, he has not received a full education and is missing much of the joy of living. But nature education does not merely give present enjoyment and recreation; it also promotes the conservation of our great natural inheritance and assists in repairing, where possible, the damage already done. It is an education which should reach everyone; it should continue from childhood until old age and as long as the race exists.

SCHOOL SCIENCE AND MATHEMATICS wants to assist in promoting this great phase of science education. As a first step we have opened a department of Nature Study and have induced one of the foremost leaders in nature education to accept the editorship. We are very happy to present our Nature Study Editor, Professor E. Laurence Palmer of Cornell University and the American Nature Association.

Dr. Palmer has had a rich experience in all activities that produce leadership in this field. He is a graduate of Cortland (N. Y.) State Normal School, holds three degrees, A. B., M. A., and Ph. D. from Cornell University, and has done graduate work at Columbia. His teaching experience includes work at Iowa State Teachers College, University of California at Los Angeles, Iowa Agricultural College, Utah Agricultural College, and Cornell University. He has held executive offices in numerous juvenile organizations such as

the Boy Scouts of America, and is now doing important work in training leaders for such work. As a writer he is well known as the author of "The Field Book of Nature Study" and co-author with Arthur Newton Pack of "The Nature Almanac" 1927 edition and also 1930 edition. The *Cornell Rural School Leaflet* is in his charge and he is Director of Nature Education, *Nature Magazine.* He has contributed frequently to educational journals and is now your editor in charge of Nature Study and Conservation.

MR. JAMES H. SMITH ON THE HONOR ROLL

After thirty-one years in Austin High School, Chicago, Mr. James H. Smith, Assistant Principal of the Day School and Principal of the Evening School, retired from active service at the end of the Fall semester. Mr. Smith graduated from Oberlin College in 1887, and did graduate work at the University of Michigan and Johns Hopkins. Before going to Austin he spent ten years as Professor of Geography and Geology at Baldwin College, Berea, Ohio. He was a charter member of the Central Association of Science and Mathematics Teachers, President of the Association in 1909 and 1910, and has always been an active member. He was a member of other scientific societies including the American Association for the Advancement of Science, the Chicago Geography Society, and the National Council of Geography Teachers. He was active in community affairs, including church, Y. M. C. A., and business men's organizations.

There is very little in Mr. Smith's appearance to indicate that he has reached the age of three-score-and-ten, and his retirement means to Chicago the loss of one of her best teachers and administrators, but it gives to other educational fields an active worker.

HERBERT CLARKE ARMS

Herbert Clarke Arms, for twenty-eight years associated with the Central Scientific Company, manufacturers of laboratory apparatus and supplies in Chicago, died suddenly on January 22, 1932, at his home in Evanston, Ill. During twenty-five years of this period he was vice president and at the time of his death was serving as vice chairman of the Executive Committee. Mr. Arms was particularly interested in the development of the reagent chemicals and the export divisions of the business. He was well known in Chicago chemical circles, being a founder member of the Chicago Drug and Chemical Association of which he was president in 1924.

Mr. Arms was born in Dubuque, Iowa, on March 3, 1871, and was graduated from the University of Illinois in 1895. He was active in the affairs of the Sigma Chi fraternity, of which he was national president in 1927. Mr. Arms was prominent in various social activities and among his clubs were the University Club and the Lake Shore Athletic Club of Chicago, the Evanston Club and the Evanston Country Club of Evanston, Ill., and the Skokie Country Club of Glencoe, Ill.

OUR BIOLOGY EDITOR HONORED AT THE CRESCENT CITY

At the recent meeting of the A. A. A. S. at New Orleans Biology Editor, Jerome Isenbarger, Crane Junior College, Chicago, was elected a fellow of the Association. This honor comes to him for meritorious work in research in plant physiology and for leadership in teaching this subject and training teachers of junior high school science. For a number of years he has been in charge of the work in the teaching of general science at Lewis Institute, Chicago, and was chairman of the science section of the Curriculum Committee of the Chicago City Schools.

WISCONSIN STUDIES SCIENCE TEACHING

The Wisconsin State Teachers Association has appointed a committee to study the status of science teaching in the state, and has set aside funds for all the necessary expenses of the committee. Editor Ira C. Davis, Assistant Professor in the Teaching of General Science, the University of Wisconsin, is chairman of the committee. The primary aim is to improve science teaching in Wisconsin. Professor Davis is interested in finding out what similar projects are being carried on in other states and will welcome correspondence on the subject.

This movement is indeed commendable and worthy of attention. While other states are curtailing educational expenditures even to the extent of eliminating essential activities, Wisconsin is appropriating funds for new investigations, and furnishing the committee money for postage, traveling expenses and printing. Hats off to the Badger state.

NEW BIOLOGY LABORATORIES OPENED AT HARVARD UNIVERSITY

Sculptures in an unusual medium, the face of an ordinary brick wall, greet the visitor to the new biological laboratories of Harvard University, which have just been opened for use. The buildings were put up first, and then their brick walls carved with spirited reliefs of plant and animal life, just as though they were stone. One of the most striking of the reliefs represents a troop of elephants.

The total working space in the new laboratories is 145,361 square feet, or approximately three and one-third acres. In addition to the most modern equipment for work in all the botanical and zoological sciences, there are two large greenhouses, each 110 feet long, on the upper floor of the central wing.—*Science Service.*

HIGHEST VOLTAGE SUBMARINE CABLE TO GO UNDER COLUMBIA RIVER

The highest voltage transmission line to carry electricity under water will soon be laid beneath the Columbia River and Oregon Slough to bring power to this city.

Three cables rated at 115,000 volts will be buried six feet under the river bottom. The distance to be covered under the Columbia is 3,700 feet and under the Slough, 1,515 feet.

In this installation, oil-filled cable will be used for the first time to make a submarine crossing. Each of the three parallel cables will weigh nearly fifty tons and their total length will be more than three miles.—*Science Service.*

THE NEW COURSES IN HIGH SCHOOL CHEMISTRY

By CLARENCE P. STEVENS,
Santa Barbara, California.

The status of the newer courses in high school chemistry is reported with reference to the extent of their use, guiding principles, objectives, difficulties encountered, and the organization which seems best adapted to optimum outcomes. The study was based on an examination of the professional literature and replies from queries sent to 235 representative high schools of the United States.*

From time to time, during the past six years, a number of articles have appeared in the literature relative to the newer courses in chemistry, variously designated as pandemic, cultural, and orientation courses. The desire to know something of the extent to which these courses have been put into practice; something of the present aims, content, and technique of presentation, and the extent to which they had proved successful in other schools, prompted the writer to make a study of the problem during the past year. This article is presented with the thought that some of the results of the study may be of value to the many other high school teachers working along the same lines.

THE GENESIS OF THE PANDEMIC MOVEMENT

There is substantial agreement among critics that the traditional courses in high school chemistry are inadequate to meet our present educational needs. Various ideas have been formulated in an effort to remedy the situation.

Banroft (1) announced "pandemic chemistry" in 1926, conceived according to the definition of the term as "pertaining to all people." The choice of the word "pandemic" is open to some objection as it may be construed to include professional or pre-professional chemistry. Therefore, perhaps "laical" would be a better term; that is, chemistry for the layman as distinguished from that for the professional chemist. In this article the term "pandemic" is used in the larger sense of the word.

EXTENT OF THE MOVEMENT

The writer sent out 235 questionnaires to representative

*This article is based on an investigation made by the writer under the direction of Professor Frederick J. Weersing, School of Education, University of Southern California.

high schools and a few colleges of the United States. From the high school teachers' reports, it appears that the course offered with the most frequency in the public high schools of the United States is no longer the traditional pre-professional or college preparatory course but a course partly of that nature and in part pandemic and which the writer has designated as "quasi-pandemic" in Table I. There is, of course at present, no sharp dividing line between these courses. The number of teachers reporting quasi-pandemic courses was 43 which constituted about 48.3 per cent of the courses offered in chemistry. The college preparatory course was offered with slightly less frequency, the number of teachers reporting this type of course being 42 which constituted about 47.2 per cent of the whole. Four of the high schools reported strictly pandemic courses which constituted about 4.5 per cent of the whole. The latter group did not include a few vocational courses which might possibly have been so classified. It would seem, from these reports, that the trend for the immediate future is toward middle ground between the old college preparatory or pre-professional and a pandemic or laical course.

TABLE I

NUMBER AND PERCENTAGE OF HIGH SCHOOLS OFFERING THE THREE TYPES OF COURSES

High schools teaching:		
College preparatory	42	47.2%
Quasi-pandemic	43	48.3%
Pandemic	4	4.5%

TEXTS

No strictly pandemic texts, designed for high school use, were reported but many teachers were omitting parts from current texts and substituting other material. One rather widely used high school text has been recently revised and enlarged so that it may be used for various types of courses by omitting certain parts. Two new pandemic texts for colleges were reported, namely those by Dr. Sampey (2) and Dr. Timm (3).

EVALUATION OF PANDEMIC COURSES

Naturally an evaluation of the new courses is not an easy matter. It was considered worth while, however, to use the

rather imperfect methods we have, even though far from conclusive. In the first place it was considered that persons who had taught both types of courses were in the best position to judge their relative merits. Thirty-two teachers who had tried both types of courses reported. Of these twenty-eight felt that the pandemic or part pandemic courses were decidedly better than the traditional chemistry course and four replied in the negative. The number of replies is not large enough to have accurate validity for the whole United States, however, it is important, when considered with the past status of chemistry, to show the trend.

OBJECTIVES

After a careful analysis of the literature, the various aims were sorted into groups which were classified as nearly as possible under the same general headings. The writer is not unaware that in the following rather inclusive list there is some overlapping.

I. To train the pupil to recognize and appreciate the merit of scientific methods of thinking and, in so far as possible, to apply this method, not only to scientific matter, but to all problems.

II. To establish an appreciation of modern chemical achievements and show how chemistry, often working thru other sciences, has re-created our civilization.

III. To satisfy the normal interest in the materials and forces of nature; to show the scientific explanation of natural phenomena in order that truth may prevail instead of superstition.

IV. To show the importance of scientific research and stimulate the spirit of investigation and invention; to encourage independent thinking thru such devices as the project method.

V. To secure the interest and motivation essential to adequate learning thru the study of things as well as words and thru the satisfaction of creative work; to foster interest in chemistry.

VI. To provide a fund of useful information useful in vocations or in pre-vocational training.

VII. To promote the use and development of chemistry for the betterment of humanity by enlisting the pupils and public in the problems of chemistry.

VIII. To teach "consumer values" such as the intelligent use of common chemicals used for medicine, sanitation, insecticides, poisons, and for many other purposes.

IX. To secure optimum results thru the use of subject matter within the comprehension of all normal pupils and which gives a reasonable return in educational achievement for the time expended.

X. To give such training as will result in respect by the public for the work of recognized scientific experts.

The above aims are ranked in the order of their relative importance as estimated by the chemistry teachers reporting.

Guiding Principles for Pandemic Chemistry

In the queries already referred to, the teachers were asked to write in a list of guiding principles for pandemic chemistry. The replies were segregated and it was found that they could be classified substantially as follows:

1. Show the dependence of our civilization on chemistry in (a) the promotion of health, (b) national defense, (c) in the home, (d) in our pleasures, (e) in agriculture, (f) in medicine, (g) in industry, and (h) in the cultural aspects of life.
2. Should be pupil centered instead of subject centered.
3. The difficulty and quantity should be such as to require only the best efforts of normal high school students. This implies the elimination, in general, of the parts of chemistry which have been shown by Dr. Powers (4) and others to be an unprofitable use of time in high school.
4. Less emphasis on laboratory technique (as those who are not to become chemists have little need for it) or the substitution of demonstration experiments in place of the socalled "cook book" type of laboratory work, especially as the former has been shown to be of equal or better value and is less expensive.
5. Should not do violence to the laws of modern psychology.

Difficulties Encountered With Pandemic Courses

Some of the difficulties mentioned with the most frequency were the habits and training of the teacher make it difficult for him to adjust himself to the new content and methods of instruction; industrial plants are not available for visitation in many localities; requires more time and skill on part of the instructor; scientific method not so well developed; lack of trained teachers; some colleges will not accept credit for the courses; and the introduction of pandemic or laical courses is hindered if not precluded by several subjects required for college entrance and which tend to crowd out science courses which are as yet elective. The difficulty of Latin and algebra for ninth grade pupils causes large numbers of students to drop out of school, due to the high percentage of failures, before they have a chance to take pandemic chemistry or other science courses. Quoting a recent article (5) we concur that "Because she takes up so much room and clings to such improbable claims, and does it with such complacent conceit, give Mother Latin the leather medal."

Increasing the Time Devoted to Chemistry

Several types of organization for different sections of chemistry classes were reported by the larger high schools, a discussion of which is beyond the scope of this article. In general, however, it would seem that the time devoted to

chemistry in high school might well be extended to a year and one-half or two years. Such courses might well begin with a year of laical or pandemic chemistry offered in the tenth grade, to be followed by an elective semester of the more difficult parts of college preparatory chemistry and a semester of qualitative analysis in the eleventh grade. In fact several schools have been offering a third semester of chemistry as qualitative analysis for some time (6).

Several reasons tend to justify an increase of the time exposure for chemistry. In the first place, the content and difficulty of high school chemistry has increased to such an extent in recent years that it is next to impossible to teach it thoroughly in one year (7). The National Research Council of Washington, D. C., has recently reported the number of doctorates conferred by sixty-three American universities at their 1930 convocations. There are three new "doctors of philosophy" in chemistry to one of the nearest rival —zoology. Not only is this true but chemistry has led the field since the beginning of the present century (8). The outstanding importance of chemistry is also indicated by the fact that chemists constitute the largest body of professional scientists in the United States, if not in the world (9). Considering the fact that these chemists usually represent larger groups engaged in useful activities in our national life and that the aims of secondary education must be interpreted in terms of the activities in which individuals may be expected normally to participate, it necessarily follows that the outstanding importance of chemistry for educational purposes should not be ignored in the construction of a high school curriculum.

While we do not state this as an established fact, the writer believes that the majority of the scientists will concur in the statement that chemistry is par excellence the experimental science and best adapted for the teaching of scientific methods of thought.

The very fact that a lay movement has appeared within the last decade designed to teach the public something about chemistry indicates that the schools have failed to provide such instruction. We are greatly indebted to Francis P. Garvan and those associated with him in the Chemical Foundation for the most important contribution made,

within the last decade, to chemical education, if not to all education, in the United States.

Finally, the teachers reporting were, by a large majority, in favor of devoting more time to chemistry in high school as indicated by their replies to the queries already mentioned above.

Some of the findings and recommendations based on the study follow:

1. The course offered with the most frequency in the high school chemistry classes of the United States in 1930-31 seems to have been a course in part laical or pandemic and in part college preparatory or pre-professional. The college preparatory course was offered with slightly less frequency. During the past five years a full pandemic course appeared and was offered in approximately 4.5 per cent of the high schools.
2. Chemistry (for all students) is hindered if not precluded by several obsolete subjects required for college entrance and which tend to crowd out science courses which are as yet elective.
3. Some of the courses called "pandemic chemistry" were of little relative value due largely to the fact that the teachers were not conversant with the aims, content, and methods of pandemic courses.
4. The trend for the immediate future seems to be in the direction of a course covering middle ground between the old pre-professional course and pandemic chemistry.
5. At least three, and preferably four, semesters should be devoted to chemistry in high school.
6. Laical or pandemic chemistry may be taught in the tenth grade and, if properly presented, may be a required course. This may be followed by pre-professional chemistry as an elective in the eleventh grade. For schools offering only one course, a course in part pandemic and in part pre-professional is indicated.
7. Pandemic chemistry, if properly presented, has large values for the realization of aims growing out of social and industrial progress and modern conceptions of education in a democracy. However, more study and experimentation is needed to work out the content and organization which will give optimum outcomes for various localities and different groups of students.

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THE SYSTEMATIC DEVELOPMENT OF LEARNING UNITS IN GENERAL SCIENCE

By John C. Mayfield,

University High School, University of Chicago

I. The Selection of Units

In selecting the units to be taught in the general science course of the University High School the instructors in charge of the course have chosen units which, in their judgment, would be most helpful in bringing about the adaptation of their pupils to life. The methods of selection which they have used cannot be described as completely objective; they have surveyed the field in the light of their own knowledge of it with a view to meeting the needs of the particular group of pupils which is to be taught, and have chosen the units of learning which they believe to be most useful. Due weight has been given to the opinions of textbook writers and other curriculum makers. Each selection has been tested by the criteria suggested by Morrison in defining the science-type unit.[1] First, the question was raised, does the intended unit contain a unifying concept which the pupil can acquire through experiences provided in science classes? Second, is the unit significant in that it will clarify the pupil's insight into his surroundings and thus make a worthwhile addition to his intellectual equipment? Third, is the unit comprehensive enough to explain a major phase of the pupil's environment and to require mental exertion and organization on his part?

Some units selected according to the preceding criteria alone prove impractical. For example, a unit may be so comprehensive and difficult as to take an unreasonable amount of time when the attempt is made to teach it at the intended level. "The Weather," as a unit, is certainly comprehensive and significant. Yet if the complicated interrelationships of the weather are taught to sixth-grade pupils the unit is too difficult and time-consuming. It is even doubtful if the unit can be economically taught at the freshman level where it is now placed. On the other hand, some so-called units appear to be so easily understood, insofar as they are essential to general education, that they

[1]Henry C. Morrison, *The Practice of Teaching in the Secondary School*, pp. 23-24, 182. Chicago: University of Chicago Press, 1931 (revised).

do not need to be taught. The average person can acquire by reading and observation as thorough an understanding of "Building Materials" as may be needed for the practical purposes of his life, especially if he has made the more fundamental adjustments which are acquired in an introductory study of science. Thus, the only adequate basis on which a unit for a general science course can be selected is the broad recognition of five criteria, *the presence of a really unifying concept, significance, comprehensiveness, economy of mastery at the intended level,* and *need for teaching.*

Correlation of the freshman science course with science courses of the University Elementary School and with the health and home economics course to eliminate duplication has further aided in deciding which units should be taught and in determining what knowledge might be assumed in the pupils who come to the course. For some time the general science course included a unit on "Food." Experience with the unit indicated that the pupils were acquiring the desired conceptions in other courses. That unit was, therefore, omitted from the course.

II. The Determination of Unit Objectives and the Organization of Teaching Material

Each unit which is to be taught is worked out and refined by a systematic procedure which includes the following steps:

(1) Preparation of a *unit-analysis,* in which are stated (a) the main understanding or *unit-concept* which is the intellectual objective of the unit, (b) the secondary concepts or *learning-elements* which seem essential to the attainment of the unit objective, and (c) the more important *contributing ideas* by which the teacher expects to develop the learning-elements in the minds of the pupils.

(2) Formulation of a tentative *pretest,* of a *guide for study,* and of *unit tests* on the basis of the unit-analysis.

(3) Trial of the tentative guide for study and tests in the classroom.

(4) Examination of the results of the teaching.

(5) Criticism and revision of the unit-analysis in the light of the results.

(6) Criticism and revision of the tentative guide for study and tests.

If the desired intellectual result of the teaching of a unit is properly conceived it can, as a rule, be summarized in a single comprehensive sentence, and the attempt to formulate such a sentence aids the teacher in clarifying and defining his objective. The central concept of the unit, "Heating and Ventilating Our Buildings," has been stated as follows: *We are enabled to live comfortably in an unfavorable climate by applying natural laws to the creation of a favorable artificial climate inside our buildings.*

Each unit is further defined and limited by an attempt to determine the steps by which the pupil must attain its mastery and to state the secondary concepts or learning-elements which each pupil must have before he can acquire the unit-concept. The sole criterion for the adoption of any step or learning-element is its indispensability in the attainment of the unit objective. One does not try to see how many ideas can be connected with the unit title but how few are necessary, when properly integrated, to produce the unit concept. Thus the unit-analysis in its simplest form, states a core of essential ideas which the instructor expects each pupil to get from the study of the unit.

The following have been selected by the general science instructors of University High School as the learning-elements of the unit, "Heating and Ventilating Our Buildings":

A. Heat for our buildings is usually secured by the rapid oxidation or burning of fuel. (As a rule, the presence of this learning-element may be assumed from the earlier experience of the pupils, especially since a study of "The Nature and Control of Fire" precedes this unit.)

B. Heat moves from a fire toward places of lower temperature in three different ways.

C. Heating devices and heating systems use the three methods of heat transfer to carry heat from a fire to the rooms of our buildings.

D. Comfort requires that we attend to certain atmospheric conditions other than temperature.

It is recognized that while a pupil is arriving at the es-

sential understandings such as are illustrated above, he will gather from his experiences a considerable amount of non-essential factual knowledge. Such incidental or marginal learning will vary in nature and amount from pupil to pupil. The brighter and more rapid workers will learn an almost unbelievable amount while the slower members of the class will carry away a minimum store of information. Since the memorization of many facts is not the teacher's chief concern in the teaching of the unit he will not be blinded by their interest and supposed value. He will, indeed, stimulate the growth of factual knowledge; some of it he will find useful as assimilative material for the essential concepts on which his attention is to be centered, but much of it will ever remain in the margin of his attention.

The same principles which guide the teacher in choosing the learning-elements of a unit are used in selecting the essential contributing ideas and expanding the unit-analysis still further. At some point the process of selection reaches variable experiences and knowledge. That is, a minor generalization is reached which may be illustrated or explained by any one of a number of instances or examples. As indicated above, such variable material provides the tools which the teacher may use to build up the concept which he is seeking to form in the mind of the pupil. He will select and use little or much of it as the case may require, but it is the means and not the end.

The analysis of a unit is carefully stated and preserved as a guide for the further preparation and teaching of the unit. It is usually stated in simple terms but conciseness and clarity may, at times, indicate the use of language which general science pupils would not readily understand. Since the analysis is only for the use of the teacher technical wording of some points need cause no concern.

A complete tentative analysis that was made up for the unit on heating and ventilation follows:

The Unit-Concept

We are enabled to live comfortably in an unfavorable climate by applying natural laws to the creation of a favorable artificial climate inside our buildings.

Learning-Elements and Important Contributing Ideas

A. Heat for our buildings is usually secured from the rapid oxidation or burning of fuel. (The presence of this learning-element may usually be assumed.)

B. Heat travels from a fire toward places of lower temperature.
   1. It may be thrown through space as radiant energy—by radiation.
   2. It may go from particle to particle—by conduction.
   3. It may be carried by the movement of materials—by convection.

C. Heating devices use the three methods of heat transfer to carry heat from a fire to the rooms of our buildings.
   1. Fireplaces depend largely on radiation.
   2. Stoves produce convection currents in the room in addition to radiation.
   3. Hot-air furnaces depend on natural or forced currents of air to carry the heat from furnace to room through large ducts.
   4. Hot-water systems employ convection currents of water in pipes.
   5. Steam systems utilize the heat and pressure of steam.

D. Comfort requires that we attend to certain atmospheric conditions other than temperature.
   1. The correct degree of humidity should be maintained.
   2. The air should circulate.
   3. Some fresh air should be supplied.

With the analysis of the unit at hand, an exploration test is prepared. The purpose of the test is to find out which pupils, if any, have already acquired an understanding of the unit or any of its elements. Experience may indicate that in teaching some units the exploration test is a waste of time and that, if a written exploration is used at all, it should be designed principally for motivation and preparation in the form of "preliminary exercises." The questions of an effective test will produce responses which definitely reveal the state of a pupil's understanding of the complete unit or of elements of the unit. When the responses are satisfactory the pupil may be excused from the assignments which he does not need. Sometimes the evidence will be inconclusive and the exploration test will serve only to indicate those few pupils who should be more thoroughly tested in conference to find whether they may be released from any of the regular work of the unit. An exploration test devised for the heating and ventilation unit follows. For convenience in diagnosing the pupils' knowledge the test is divided into four major sections which correspond with the four learning-elements of the unit.

HEATING AND VENTILATING OUR BUILDINGS

Exploration Test

Class..... Date..... Name.....

Instructions.—Fill in the blanks above when you first receive this sheet. Then answer the questions as well as you can on a separate sheet of paper. At the end of the hour hand in this question sheet and your answers.

A.

1. How do we usually get the heat with which to heat a building during the winter?

B.

2. At what temperature should the air of this room be kept?
3. (a) Name the three methods by which heat moves from one place to another.
   (b) Give three illustrations or examples of each of the three methods, a total of nine examples.

C.

4. Make a list of the different types of heating devices and heating systems now in use in this country.
5. Explain as well as you can the heating plant which heats your home. Use diagrams to make your explanations clear.
6. Give two important differences between the heating system of your home and each of two other types of heating systems, a total of four differences.
7. What becomes of the steam which is condensed in the radiators of a steam heating system?
8. What makes the hot water go to the radiators in a hot-water heating system?

D.

9. In addition to warming the air in our buildings what conditions do scientists believe must be provided to keep them most comfortable?
10. What do we mean by humidity?
11. Is carbon dioxide poisonous? Why do you think so?
12. Why does a crowded room become uncomfortable? Answer this question with care. A superficial answer will not be satisfactory.

The use of an exploration test is justified by the amount of voluntary advance work which it stimulates on the part of able and interested pupils and by the improved attitude produced when pupils are excused from the study of material which they already understand. Some pupils finding that the teacher will really release them from some of the assignments will go far toward mastering a unit before the exploration test is given.

Ordinarily the teacher introduces a unit to the class with an overview or presentation. In such a talk some teachers attempt to explain the entire unit and to state all the ideas which the pupils will be expected to gather from its study. If such a lecture were really effective there would be no reason for spending weeks of study on a unit. A more successful type of presentation orients and motivates the class by pointing out the importance of the unit, by connecting it with the preceding experiences of the pupils, by giving a clear idea of its extent, and by telling of its interesting aspects.

The work of making the unit analysis will have prepared the teacher well for such a talk. He will, however, need to think the unit through again very carefully, searching out significant and interesting applications, illustrations, and bits of information which he can use in interesting the

pupils in the unit and in making them see what it really is.

A mimeographed Guide for Study is next prepared for use in directing the efforts of the pupils to master the unit. One type of guide has very brief topical outlines to indicate the essential ideas of the unit. Following the outline for each element are the study references and one or more problem exercises based on that element. Another type omits the topical points, gives general references or none, and has the problem exercises less carefully arranged according to the unit elements. The latter type of guide requires that the pupils give more thought to the organization of the material and is appropriate later in the course.

In building the tentative guide sheet it is necessary to decide on the assimilative material, such as reading, demonstrations, and excursions, which will be used. Then appropriate references are selected and a series of problem exercises is provided to stimulate study and thought.

The successful study of a unit is as largely dependent on the study exercises as on the assimilative material used. To be most successful their solution should require the mastery of essential ideas of the unit and not be mere busy work. They should stimulate thought instead of the copying of information from books, yet they should not be too difficult. They should break the mastery of the unit into parts manageable by the pupil, yet mastery should not require too many of them. They should be inherently interesting for the pupils. Easily and quickly corrected forms of reports are desirable in order to lessen the teacher's burden of written work. Exercises which do not approximate these standards will result in waste of time and effort by teacher and pupils.

A tentative Guide for Study used in teaching the unit, "Heating and Ventilating Our Buildings," follows:

HEATING AND VENTILATING OUR BUILDINGS

Guide for Study

A reasonable number of exercises cannot be devised to cover all the points on heating and ventilation. You should, therefore, try to learn as much as possible while we study the unit.

Good material on this unit is to be found in many books to which there is no reference in this guide sheet. (Tower and Lunt, *The Science of Common Things*, contains directions for a number of simple experiments in which you will be interested. See pages 145-218.)

A. *Fire as a Source of Heat.*

1. The stored energy of fuel.

2. How energy is released.

References:

Pieper and Beauchamp, *Everyday Problems in Science*, 235-42.[2]
Barber, *First Course in General Science*, 77-79, 80.
Bowden, *General Science*, 20-21.
Clement, Collister and Thurston, *Our Surroundings*, 81-83.
Hodgdon, *Elementary General Science*, 137.
Wood and Carpenter, *Our Environment: How We Use and Control It*, 121.

Exercise 1. When one builds a fire he places the easily kindled material at the bottom and then puts on larger and larger pieces. He then lights a match and starts the kindling to burning. In a little while there is a great fire which gives off much more heat than was given off by the match. Where did this larger amount of heat come from? Explain carefully.

B. *The Transfer of Heat from Place to Place.*

1. Radiation and examples.
2. Conduction and examples.
3. Convection and examples.

References:

Pieper and Beauchamp,[3] 56-57, 176-77, 230-31, 261, 262-68.
Barber, 113-22.
Bowden, 44-56.
Clement, Collister, and Thurston, 102-104.
Hodgdon, 87-103.
Hunter and Whitman, *Civic Science in the Home*, 185-86.
Pamphlet, *Keeping the Home Fires Burning*, 9-10.
Webb and Didcoct, *Early Steps in Science*, 64-70.
Wood and Carpenter, 233-237.

Exercise 2. List ten common instances of the transfer of heat by radiation, ten of its transfer by conduction, and ten of its transfer by convection.

Exercise 3. Prove that warmer parts of fluids (gases and liquids) are pushed up by the cooler parts and do not "rise" of their own accord as people commonly suppose. (Prepare a brief outline of your proof and have the instructor approve it before writing it out.)

C. *The Transfer of Heat by Common Heating Devices.*

1. The structure of common heating devices.
   a. Fireplace.
   b. Stove.
   c. Hot air furnace.
   d. Hot water system
   e. Steam system.
2. How each device applies the methods of heat transfer to its work of heating rooms.

References:

Pieper and Beauchamp, 261, 264-71.
Barber, 120-145.
Bowden, 77-84.
Hodgdon, 104-111.
Hunter and Whitman, 188-198.
Pamphlet, *Keeping the Home Fires Burning*, 11-26.
Webb and Didcoct, 72-75.
Wood and Carpenter, 218-219, 226-243.

Exercise 4. Trace the heat from its source to the air of a room through each of the devices listed under *C, 1*, above. Name the methods of transfer in each place where they occur. Thus, in explaining how the heat reaches the room from the fire in a hot air furnace one might say, "The heat energy is released from the fuel in the fire pot by its rapid oxidation. It is transferred to the iron walls of the fire

[2]All references are to page numbers.

[3]Titles not given are the same as in preceding references.

pot by conduction and radiation, passes through the iron and into the air surrounding it by conduction, etc. . . ."

Exercise 5. Draw carefully and label fully a simple diagram of the heating system used in your home. Explain how it works. (Refer to your account in *Exercise 4* if you wish.)

D. *Keeping the Warm Air Comfortable.*

1. Providing the correct degree of humidity.
2. Supplying a sufficient amount of fresh air.
3. Maintaining circulation.

References:

Pieper and Beauchamp, 272-76.
Barber, 311-342.
Bowden, 163-168.
Hodgdon, 171-181.
Pamphlet, *Keeping the Home Fires Burning*, 27-30.
Ritchie, *Sanitation and Physiology*, Part II, 178-185.
Webb and Didcoct, 77-84, 552-553.
Wood and Carpenter, 141-147, 456-458.

Exercise 6. Read carefully the reference in Pieper and Beauchamp studying the diagrams and pictures and make a list of the important points which are new to you.

Exercise 7. Why is the air indoors in the winter time ordinarily so much less humid than the air outdoors?

Exercise 8. Tell how fresh air is supplied to rooms heated by each of the main types of heating devices.

Topics for Supplementary Study

Radiation (uses, disadvantages, control, etc.).
Convection Currents and Their Importance in Heating.
Convection Currents and Their Significance in Weather.
Conduction in Different Substances.
Heat Insulation (in buildings, in steam and hot-water lines, kinds of material used, etc.).
Recent Improvements in Heating Devices.
Are Oil Burners More Satisfactory Than Coal Burners in Heating Systems?
How Can the Proper Degree of Humidity Be Maintained in Our Buildings (measurement of humidity, various ways of supplying it, etc.):
The Regulation of Temperature in Our Science Room.
Thermostats.
Experiments with Heat. (See Tower and Lunt.)

Pupils retain the knowledge and methods of procedure emphasized by the study exercises much better than they do material which comes to their attention through incidental reading, explanation, or observation. Great care should be used, therefore, in devising suitable exercises. The Guide for Study given above illustrates several profitable types. *Exercise 2* requires a pupil to interpret everyday experience in terms of new ideas and gives him new and accurate terms with which to express himself. *Exercise 4* encourages systematic thought in that the pupil must construct the sequence of steps in the passage of heat from a fire to a room where it is needed. If he lacks the necessary knowledge of heating devices he must go to the reference

material for it. But even then he cannot write a satisfactory exercise by copying information into his paper. He must think out the steps called for. *Exercise 5* gives training in the use of simple diagrams to represent familiar things. *Exercise 3* was prepared to help eliminate a common erroneous interpretation of convection currents by requiring the pupil to give reasons for the correct interpretation.

When a pupil has completed his study of a unit as it has been suggested by the Guide for Study we may not assume that he has really gained the desired concept. Only when he has given good evidence through his own mental behavior are we justified in concluding that the desired learning has occurred. Of course, the best way of checking the results of teaching is to find out how the pupil reacts outside the classroom to situations connected with the unit. Such a test is manifestly impractical for use in school and some form of classroom test is commonly used to gather evidence of the pupils' understanding of the unit.

For a number of years best-answer and multiple-response tests were used in general science. There were so many instances where the evidence produced by these tests proved unreliable that they were abandoned, at least until their construction is better understood. Instead, carefully written question-type tests are now being used. The questions are based on material familiar to all the pupils and are intended to bring out responses from each one which will show whether or not he understands the unit.

Some questions in each test are based on each learning-element, some call for the recognition of the inter-relationships of the different elements, and, if they can be found, some require the use of the complete unit-concept.

Sometimes a pupil will miss several questions meant to reveal a certain learning product but will answer others in such a way as to make the reader positive that the essential ideas have been acquired. Tests for mastery cannot, therefore, be graded but must be read and judged on the basis of the understanding displayed in the test as a whole and not on the correctness of the individual answers. Furthermore, in judging test results the quality of English used by the pupil must be ignored.

It should further be said that some pupils who show great lack of skill in interpreting and answering questions are given special training. They are required to write new answers to the test questions which they have answered poorly; they are shown how to write answers, and are given other questions to answer, although their understanding of the unit itself may not be in doubt. Every pupil who makes incorrect statements in his answers to test questions is also required to correct them before his work is regarded as complete.

One of the final tests used with the unit on heating and ventilating will serve to illustrate the general plan.

HEATING AND VENTILATING OUR BUILDINGS

Unit Test—Form A

Class..... Date..... Name.....

Instructions.—Fill in the blanks above when you first receive this sheet. Then answer the questions as well as you can on a separate sheet of paper. At the end of the hour hand in this question sheet and your answers.

A.

1. What is the usual source of heat for our buildings during the winter?

B.

2. If a person had never studied science how would you explain to him the difference between radiation and conduction of heat? Be sure that your answer would make him understand. Give some illustrations to help him out.

C.

3. Convection has a part in at least three of the steps necessary in carrying heat to a room through a hot-water heating system. What are these three steps?
4. If you were building a seven-room house for your own use which method of heating would you install? Why would you choose that rather than one of the others?
5. Why do steam radiators have air vents?

D.

6. What is humidity? Why is it considered important in this unit?
7. Write a half-page paragraph which will tell the most important things about "Making a Good Artificial Climate in the Home."

The general science units which have gone through the process of development described above and are in use in University High School are:

I. Our Earth and Its Neighbors.
II. The Weather.
III. The Water Supply (including the disposal of waste water).
IV. The Nature and Control of Fire.
V. Heating and Ventilating Our Buildings.

VI. The Work of the Body.
VII. Man and Microbes.
VIII. Simple Machines and Work.
IX. Harnessing the Energy of Nature to Do Man's Work.
X. Generating and Using Electric Current.

METHODS OF TEACHING BIOLOGY

By Hazel A. McDowell,

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In those schools where biology is included in the curriculum it is taught in various ways, with varying degrees of success. On the whole, apparently it is not fulfilling its potential functions. No doubt one of the main causes for this deficiency is the manner of its presentation.

This article attempts to briefly summarize methods now in use by a representative number of teachers in the teaching of biology. It is left to the reader to interpret and evaluate these methods in the light of his own peculiar difficulties and those due to local conditions.

The plan of investigation by which the methods were collected was that of job analysis as described by C. C. Crawford.[1]

How to secure valuable contributions from biology for the student

Any subject which makes no worth while contribution to the life of the student is not worthy of the expenditure of teacher and pupil time or of the actual money it costs to teach it. Some fine contributions with various ways of securing them are:

(1) Right use of leisure time.

(a) Teach biology as an aid to efficient, enjoyable living rather than as a basis for a future college course.

(b) Use "nature study" as an introduction to a more careful study of one's environment.

(c) Introduce a new world of interest and entertainment through microscope work.

(d) Use the "collecting instinct" to establish a worth while life hobby.

(e) Organize a camera club among biology students

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[1]C. C. Crawford, *The Technique of Research in Education*, University of Southern California, Los Angeles, 1928. Chapter VIII, "Job Analysis."

and thus teach a method of study without destruction of the specimen.

(f) Assign fascinating references in the line of the pupil's special biological interests.

(2) Health.

(a) Lay a foundation for the health concept by presenting protoplasm as the greatest of life phenomena—as living clay for living buildings.

(b) Show that the type of "future house" in which we live is dependent upon present health habits.

(c) Show that health habits which now require conscious will power, will later become automatic.

(d) Study bacteria to find out how health is dependent upon the nurture of helpful bacteria and the destruction of harmful ones.

(e) Study epidemics to show how individual and public health are interdependent.

(f) Show, through study of physiology, that health is simply normal functioning of body parts.

(g) Use American Medical Association charts to expose quack doctor methods and patent medicine frauds.

(h) Experiment with various diets on white rats.

(i) Use statistics from local health department to show the advantage of preventative over the cure measures.

(3) Ethical character.

(a) Study such phenomena as assimilation, capillarity and the activity of cambium.

(b) Have poems like Joyce Kilmer's "Trees" read in connection with tree study.

(c) Use the study of reproduction from lowest to highest life forms to show man's inability to understand what life really is.

(d) Use biographies of great biologists to inspire thoughts of an unselfish life of service.

How to manage the equipment problem

Just what is needed in the way of equipment must depend upon location of school, available funds, ingenuity and executive ability of the one who requisitions it, type of course offered, and the facilities for its storage and use. Some considerations to be made concerning choice of equipment are:

(1) Location and setting of the biology unit.

(a) Locate it preferably on ground floor.

(b) Have green house and garden within easy access to the class room.

(c) Have greater floor space than for ordinary class room on account of the accommodation of special equipment.

(d) Have lighting on three sides if possible; if not, then on east and south so as to afford microscope illumination through the entire day.

(2) Class room.

(a) Teacher's desk should afford a view of the entire room and easy access to each student.

(b) Have fixed furniture consist of plenty of cupboards and shelves preferably under windows so as not to interfere with illumination or aesthetic appearance of the room.

(c) Have under window cupboards finished off with table top effect with an occasional sink which can be covered flush with top when not in use.

(d) Glass cases for exhibit of specimens should be in room or near by in the hallway.

(e) Have individual student desks supplied with enough drawers to accommodate all class sections. Have each drawer supplied with individual keys. Have keyboard for each class and have keys left in the class room.

(f) Chairs should be swivel jointed but should not bend backward.

(g) Have store room adjoining class room supplied with plenty of cases, shelves, cupboards and a table to hold apparatus when transferring it to and from the cases.

(3) Individual student equipment should include paper, texts, notebook folder, manual, microscopic slides and cover glass, drawing pencil, eraser, pen, and ink.

(4) General class equipment. Secure catalogs from the various supply houses and determine a list of equipment commensurate with needs for the particular course offered and funds available for its purchase. No one list could fit all places.

How to initiate the course

"A good start is half the battle," is true of the biology

course. Some essential pointers in insuring the right start are:

(1) Have a definite plan.

(a) Organize the course.

(b) Learn the pupil background.

(c) Make a survey of available material aids to be found in the community, the city library, the school library, science department and biology store room.

(d) Decide upon minimum essentials for the year.

(e) Divide minimum essentials into suitable units according to semesters or quarters.

(f) Have the detailed plan for entire year printed, typed or mimeographed for handy reference.

(2) Motivate the program.

(a) Interest pupils at once by having on board questions whose answers entail biological information and challenge the interest of pupils.

(b) Tactfully remove prejudice against biology.

(c) Begin with organic rather than inorganic things by calling attention to objects in the room and asking which have lived.

(d) Begin with something well liked by all, such as flowers or birds.

(e) Begin with cell study—all young folk are eager to learn to use the microscope.

(f) Begin with some type of manual work to give them something to do from the start.

(g) Avoid the humdrum and dull. Do not stay on one subject long enough to bore most of the students.

How to supplement the text

No purely textbook course could be a real biology course, so plenty of methods of supplementation must be used.

(a) Use charts on such subjects as food, health, or textiles. Secure free charts from meat packing companies, tooth paste companies, cereal manufacturers, et cetera. Other charts may be purchased from the various scientific supply houses.

(b) Secure exhibits in same ways as charts and on such topics as silk moths, flax to linen, life histories of helpful and harmful insects and plants.

(c) Use lantern and slides, the latter being selected to correspond with the topics under consideration.

(d) Use magazines such as National Geographic for illustrations and reading content.

(e) Secure models of various structures such as root tip, stems, leaf, human ear, eye, manikin, et cetera. These facilitate necessary structure study.

(f) Use newspapers whenever they contain authentic material but avoid sensational and fictitious descriptions of nature.

(g) Secure pamphlets from manufacturing concerns, the federal government, and state board of health.

(h) Use moving pictures and still pictures as illustrative material on various subjects.

(i) Secure stereoscopes with pictures for study when actual specimens are unobtainable.

(j) Use various reference books because no single author can be equally well informed on all subjects.

(k) Have pupils make special reports on various supplemental material. This saves class time.

How to manage the specimen problem

Specimens are especially necessary for the student who lacks imagination. Some useful hints concerning specimens are:

(a) Secure living specimens for class study when possible. They are more interesting to students.

(b) Avoid "circus tendencies" in studying living specimens for they defeat careful work.

(c) Dead specimens require less care than living ones but must be preserved either by drying or in preservative solutions.

How to secure specimens

(a) Train alumna to send in specimens.

(b) The teacher should bring in especially those things which will be needed for future work.

(c) Have students bring in anything which interests them. Give extra points for what they bring in.

(d) If funds permit, secure specimens from supply houses when not accessible otherwise.

(e) Have students take field trips and take them as a class for collection purposes.

(f) Exchange specimens with other schools whose different location would result in the finding of different species of flora and fauna.

How to care for specimens

(a) Furnish living ones with an environment as nearly natural as possible.

(b) Dead specimens must be preserved for one of two purposes—that of class study and dissection and for exhibit purposes. The former requires only formalin in a 3%-10% solution. If for latter purpose, a clearer solution like alcohol or alcohol and glycerine or a combination of Klotz and Carlsbad solutions. The formulae for these will be furnished by medical schools or apparatus companies.

(c) When preserving specimens by drying be sure to arrange as desired for study because when they are dried they cannot be changed without breaking.

How to study specimens

(a) Place a list of parts on the board asking students to identify them on their specimen and determine their functions.

(b) Give to students an outline including such items as habitat, description, locomotion, food and food getting, reproduction, and economic importance and they may be asked to find out these items by books and observation.

(c) Have minute study done with the aid of hand lenses, binoculars, and microscopes.

(d) Teach the anatomy and care of the microscope before teaching its manipulation.

How to secure proper methods of biology study

With such great variation in mental ability and rates of study it is impossible to make any one method fit everyone. Several things must be known concerning these variations before any adequate selection of methods could be made.

(a) Determine first the mental ability of the class by consulting the I. Q. record in the office. This is not to be considered as infallible but it helps in a general way.

(b) Test the class to learn whether they are auditory, motor, or visually minded.

(c) Determine the various rates of learning by assign-

ing the same task to all and recording the finishing time of each.

(d) Determine special aptitudes and interests of each pupil for this particular field of work by conferences.

(e) Collect methods of study which would seem best suited to the majority of the class members. Include any special methods which in the light of best current knowledge of psychology and pedagogy would fit the special cases. Ask students for any methods or devices which help them to learn more rapidly and effectively.

(f) Discard duplicates.

(g) Arrange these methods in as concise and convenient form as possible.

(h) Give a copy to each student, asking him to try them out and select the ones best suited to himself.

(i) Mail copies to parents and ask their cooperation in establishing right study habits.

(j) Have general study halls where teachers can at least insist upon an atmosphere of study.

(k) Have a departmental study hall where pupils can be helped to use methods best suited to the acquisition of scientific knowledge under the supervision of a science teacher.

(l) Evaluate the results of these methods continuously to determine their success or failure. Various kinds of standard tests can be of service here in determining the amount of content acquired by method.

(m) Evaluate ultimate gains in any way possible. This will be most difficult but the noticeable effect upon pupil's study habits and accomplishments, and the evaluations which the students themselves put upon the process will help some.

OFFICERS OF THE MATHEMATICAL ASSOCIATION OF AMERICA

At the annual meeting of the Mathematical Association of America the following officers for 1932 were elected: Vice Presidents, W. H. Bussey, of the University of Minnesota and G. C. Evans of the Rice Institute; members of the Board of Trustees (for three years), H. E. Buchanan of Tulane University, W. B. Ford of the University of Michigan, E. R. Hedrick of the University of California at Los Angeles, and W. R. Longley of Yale University.

W. D. CAIRNS, *Secretary-Treasurer.*

DEVELOPING A CONCEPT OF PROPORTION BEFORE PRESENTING THE FORMAL WORK

By T. L. Engle,
Isaac C. Elston Senior High School, Michigan City, Ind.
Drawing by Nathan H. Ranck (Student)

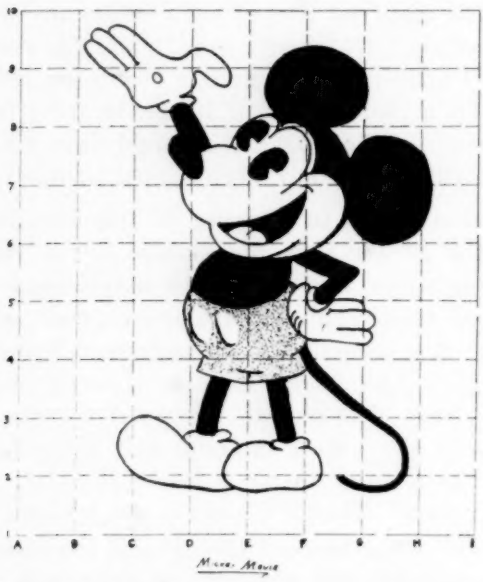
At the close of each school year it is the custom of the writer to have pupils fill out blanks telling which division of geometry they have liked best and which they have liked least, giving reasons for their answers. It is always explained to them that the answers are to be perfectly frank and that they will have no bearing on final marks, it being the wish of the teacher to improve the course on the basis of their answers. Answers are usually given with characteristic adolescent frankness. Formerly the majority gave "Proportion" as the section liked least, and "It was hard to get the idea in my head" was a typical reason. Since adopting the method described below, proportion has come to be the most popular part of the course, and objective tests show that a clear understanding of the topic has been gained. Before adopting the new method of approach it was common to have a pupil give a proportion in which a long line was to a short line as a short line was to a long line and have no sense of incorrectness. Theorems about similar triangles and lines parallel to bases of triangles were faithfully memorized, but many pupils failed to gain a true concept of proportion. Probably some other teachers have had this same difficulty. Now, by the method described below, the writer develops the concept well in advance of the formal work and absurd proportions have been practically eliminated.

One day while rushing through a famous art gallery of Europe with a typical crowd of American tourists "doing" the word's most famous paintings in an hour or so between trains, the writer suddenly almost literally ran into a geometry problem, and decided to try it out on the next geometry class under his instruction. Young artists, in copying the masterpieces, use the following method. Across a photograph of the original painting small squares are drawn. Then on the canvas which is to contain the reproduction larger squares are drawn in faint charcoal lines. Later these lines can be removed easily. By locating points on the canvas in squares corresponding to squares on the

photograph the main outlines of the picture are obtained in correct proportion. Of course, artistic skill is required for the details. Possibly master artists frown on this method of reproducing a picture, but if it can be used in copying the masterpieces of the world, we should have no compunction against using it in our high school geometry classes.

Two or three weeks in advance of the formal work on proportion the writer tells his classes about the method used in art galleries, and makes an assignment requiring that each pupil select some picture and reproduce it by that method. The first time that this assignment was made consternation reigned for a few days until some tried out the method and were shocked at the good results obtained. Since that year the writer is frequently asked throughout the course, "How soon do we come to the part where we make the pictures?" That the work is not irksome is proved by the fact that some pupils make five or six pictures when only one is required and recorded in the grade book. No time need be taken from the preparation of regular daily assignments for this work. Following this assignment it is always of interest to note the number of pictured posters, note-book covers, raincoats, and tire covers which spring up. Later in life some may have occasion to use the method in making scenes for window displays or for the stage.

For class work Sunday comic supplements and daily comic strips make excellent "masterpieces" and add a little fun to what is popularly known through prejudice as a dry subject. Of course, some pupils will choose more serious pictures from the better magazines. Simple line drawings are best, although pupils with a little artistic ability can do excellent work reproducing shaded drawings and even photographs. Squares, for example of $\frac{1}{4}$ inch, are made on the original picture, and then light pencil squares, say of $\frac{1}{2}$ inch, are drawn on a sheet of drawing paper. Lines of the original drawing are reproduced in the corresponding squares. In the illustration the squares are drawn in ink but for class work they can be put in in such light pencil lines that they will be scarcely noticeable. Even without formal training in ratio the pupils can understand what is meant by the term and indicate it on their drawings, thus building a foundation for the later formal work on ratio.



A little color added with crayon or water colors will do much to liven up the pictures. If some of these pictures are posted on bulletin boards where all of the high school pupils can see them, something can be done to create a popular opinion of mathematics as an interesting subject. Of course, pictures may be reproduced in the same size as the original, or they may be reduced in size, but enlargement has proved to be easiest for students and is by far the most popular process.

Other methods may be used to introduce the concept of proportion although the method described above has the most appeal to students. Reproductions made with a pantograph are enjoyed and help give a thorough understanding of proportion. Most pupils of this age are interested in how to make things. Mechanical and architectural drawings cut from magazines and brought in act as further examples of proportional work. Also, some are likely to be interested in photography as a hobby, and a few snap shots with their enlargements will add another contact and will help to develop an understanding of the topic.

Such a development of proportion before the formal work on the subject is taken up has three distinct advantages: (1) A keen interest in the topic is developed, (2) A

clear conception of proportion is laid down on which to build the more formal work, (3) Part of the popular prejudices against all mathematics courses as uninteresting, unpractical, bookish subjects are broken down.

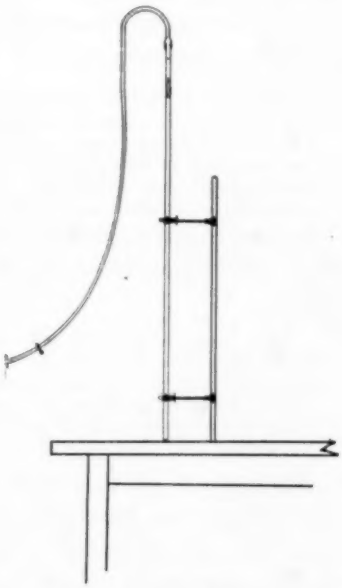
✓AN ELONGATED CARTESIAN DIVER

By F. Joseph Lorz,

Monticello Junior High School, Cleveland Heights, Cleveland, Ohio

An interesting device for illustrating the transmission of pressure beneath the surface of liquids and incidentally the principle of flotation is a Cartesian diver, the construction of which is herewith explained. The writer has set up one in which the time required to complete a dive in either direction could be varied from three to six minutes, depending on the pressure exerted on the surface.

The outside vessel is a piece of shipment length glass tubing (about five feet) with an outside diameter of twelve millimeters, one end of which has been closed by heating. It is filled to within a few centimeters of the top with water to which may be added crystals of potassium permanganate for color.



The diver is also made of glass tubing, but with an outside diameter varying from seven to eight millimeters and the length from six to eight centimeters. One end is drawn to a point, closed and the tube is inverted in the liquid in the outer vessel. By trial and error, within the variations of length and diameter mentioned, it is possible to make a diver that will float with its sealed point just above the surface of the liquid.

The end of a piece of rubber tubing about four feet long is fitted around the open end of the outer vessel. The system is then clamped upright to a lecture-table standard. An increase in the pressure on the surface of the liquid is affected by blowing into the open end of the rubber tube and the "imp" slowly sinks. This pressure is then maintained by closing the tube with a pinch-cock. Conversely, by sucking on the tube the pressure is decreased and the diver rises.

Its teaching value as a lecture table demonstration lies in its spectacularity, and in the duration of time necessary to execute a dive, during which differences of level in the vessels may be readily noted by an entire class.

SOME QUESTIONS CONCERNING THE PLACE OF SUBJECT-MATTER IN THE TEACHING OF PHYSICS

By Fred J. Mulder,
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The purpose of this paper is to raise a few questions regarding the place of subject-matter in the teaching of physics. The general trend of at least a part of the discussion will probably apply to most of the secondary school science, but being a teacher of physics, the writer will speak in terms of that subject.

The present tendency in science teaching is to find its major objectives in something aside from subject-matter itself. These objectives take the form of "training for critical thinking," "instilling the scientific attitude," "teaching the scientific method," etc. A few quotations will illustrate the point.

In the book by Brownell and Wade entitled *The Teaching of Science and the Science Teacher*, we find this statement:

> "As the facts and theories of any science are used day by day under the teacher's direction, it is not primarily for the mastery of science by scientists in embryo. Rather, the teaching purpose is to establish and maintain teaching situations in which shall be developed thinking men and women with a scientific attitude towards the affairs of life."[1]

Again in an abstract of a report on *The Curriculum in High School Science*, by F. D. Curtis, this statement appears:

> "The emphasis upon the college preparatory aim which was dominant before 1900 gave place early in this century to the practical aim in science teaching. This in turn is coming to be relegated to a position of secondary importance to the emphasis upon training boys and girls in habits and methods of scientific reasoning and thinking."[2]

Statements of this type by writers and speakers in the field of science teaching can be multiplied almost without number. Every science teacher is familiar with the trend of the argument. Let it be understood that there is no intention, here, of disagreeing either with the objectives thus stated nor the relative emphasis placed upon them as compared with other objectives. Our purpose here is to discuss what this means in terms of subject-matter in the teaching of physics.

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[1] "The Teaching of Science and the Science Teacher." H. Brownell and F. B. Wade. The Century Company, 1925.

[2] "The Curriculum in High School Science," an abstract. National Educational Association Proceedings, 1929, P. 561.

It may be of interest to recall that the search for objectives outside of subject-matter is far older than the present generation. From Plato on through the "faculty" psychology and "mental discipline" of the last generation ultimate goals of education have usually been something aside from specific attainments in particular subjects. The present tendency based, without doubt, on the more recent psychology simply perpetuates this search for objectives of education outside of subject-matter.

THE PROBLEM

Let us consider, briefly, the implications of this tendency for subject matter in physics. It would seem obvious if our main object in science teaching is to achieve scientific attitudes and habits of thinking, that one science subject will do as well as another. The general type of thinking and methods of developing the various branches of science are quite the same. Biology and chemistry were developed by much the same type of thinking and by men of the same general attitude toward truth as was physics.

Not only is this true in the natural sciences, but the social sciences—sociology, psychology, and education itself—are claiming a new body of knowledge built up by the application of this same exacting scientific method. In fact, it will not be long before almost any subject in the curriculum may claim a right to hold as objectives the inculcation of these scientific attitudes, ideals, and methods. Far from objecting to this fact, the true scientist will rejoice that the need for these objectives is becoming so generally felt and will wish them all godspeed.

This brings us, however, directly to our immediate problem. If the major purposes of teaching physics may be attained through many other types of subject-matter, then physics may only rightly maintain a place in the secondary school curriculum if it has, entirely aside from these objectives, a unique subject-matter which serves a valid and essential place in child development. A proper discussion of this problem should probably include answers to the following questions:

1. What functions may subject-matter in physics serve?
2. Which of these functions are unique for this subject?
3. Which of these unique functions are of essential value when

viewed from the standpoint of the general objectives of all education?

Before proceeding to discuss these three questions, let us point out two things: first, the intention, here, is to outline the problem rather than attempt to settle it; and second, there is no intention to put physics, in particular, on the defensive. Every other subject is in the same position in this respect and must defend itself along the same general lines.

The Purposes of Subject-Matter

First, then, what purposes may subject-matter serve in the teaching of physics?

The following list contains those purposes of subject-matter which are usually mentioned. It is obviously not exhaustive. It would also appear to be obvious that no one kind of subject-matter will be valuable for each purpose but only when it is chosen for that particular end will it necessarily further any particular purpose.

1. Subject-matter may assist in the teaching of the scientific attitude and the scientific method.
2. Subject-matter may assist in building up an appreciation of the great achievements of science and of the major scientists who have contributed to this achievement.
3. Subject-matter may reveal the general cause and effect relationship in nature and assist in overcoming superstitious dread and outworn beliefs.
4. Subject-matter may directly prepare for further work in science and the vocations built on science.
5. Subject-matter may reveal the nature, requirements, and possibilities of the vocations built on science, for the purpose of guidance in the choice of a vocation.
6. Subject-matter may directly assist one to read intelligently the science articles in the daily newspapers and magazines.
7. Subject-matter may satisfy natural interests in the forces of nature with which we are surrounded and with which we must deal.
8. Subject-matter may be directly useful in dealing with those appliances and principles which science has developed which are useful in making for comfort and convenience in home and community.
9. Subject-matter may assist in giving an insight into the nature and organization of the environment, making of it an organized whole instead of a lot of disjointed parts.

The second point in the statement of our problem, raised the question as to which of these subject-matter purposes might be considered unique for the subject of physics. Let us, briefly, examine each of them in the light of this question.

The first three objectives fall into somewhat the same category. We have already pointed out that other subjects than physics may achieve the purpose of inculcating scientific attitudes, ideals, and methods. Appreciations of the great benefits of science and of the cause and effect relationship may possibly also be instilled by subject-matter in other fields than physics.

It might be argued, of course, that all of this can be better done in physics (or at least in the physical sciences), where the materials are more objective and the principles more mathematically precise than in other subjects of the high school curriculum. And in truth, the fact that it was in this field that the experimental methods of the scientist were first fully developed and vindicated, may not lightly be ignored in building a curriculum which pretends to bring the child into contact with the major achievements of mankind.

This is in danger of reducing to mere argument, however, supported mainly by the unavoidable bias one acquires from continual contact with a particular subject. The intention here is not to argue but to bring out the salient points for consideration.

We should also note in this connection that the extent to which attitudes and appreciations result from teaching method, teacher personality, and many other factors, rather than from a particular type of subject-matter, remains to be determined.

Our fourth subject-matter objective deals with vocational preparation. The secondary school has, of course, given up as its major aim the preparation for college. It retains, however, as one of its aims the preparation for vocation. It would seem obvious, therefore, that where pupils have chosen one of the vocations demanding further work in science, the secondary school should make some provision for the necessary fundamental training, and where the demands are in physics, no other subject as now constituted can offer the necessary material. The question may be raised as to whether present college requirements in this field are of greatest ultimate value to even those pupils who plan on some sort of a scientific career. But this is aside from the present issue. As the matter stands now, we may assume that a certain amount of college preparatory phys-

ics is justifiable for groups planning on advanced work in science.

The question as to how much a high school course in physics should stress the vocational guidance function is as yet unsettled. It may eventually become one of the major purposes of the course. On the other hand, it may be taken care of, as it now is in some schools, by separate guidance courses and so be purely an incidental matter in the physics course. As yet the guidance problem has called forth no separate set of material to be incorporated into physics texts.

Whether it be a major objective or not, it has been shown that a large number of the science terms in the daily press are from the field of physics. To what extent this need can be filled by other courses will depend on the number and nature of these terms and amount of basic knowledge of physical theory needed to make them intelligible.

The satisfaction of interests in natural phenomena which constitutes our seventh objective, has been accepted as one of the major functions of a course in science, and here physics has a body of material which can hardly be duplicated by any other subject. Other sciences may satisfy interests in some phases of Nature's varied expressions, but they do not duplicate the field covered by physics. If this purpose of subject-matter proves to be an enduring, essential objective in comparison with other objectives of secondary education, then we have here a body of subject-matter which is unique and worth while.

In somewhat the same category are the last two subject-matter purposes. No other subject can furnish the material for understanding the many useful and interesting devices which now serve mankind. Electric lights, the automobile, radio and television, to mention just a few, are products of applied physics. And also, only through physics can we obtain a complete insight into the organization of the Universe—the nature of matter and electricity, energy transformations, etc. These types of subject-matter certainly are not duplicated by other fields.

From this cursory examination it would appear that subject-matter in physics undoubtedly serves some general purposes of education which are not served by other courses—

particularly in furnishing material which will explain and organize the environment, be directly useful in explaining and manipulating many common appliances, and in satisfying interests in natural phenomena. That it is also a preparatory subject for some vocations is, of course, unquestioned.

Our next question was concerned with the importance of these unique subject-matter purposes in serving the general objectives of education. Here we have obviously very little on which to base an argument. Whether we turn to the "Seven Cardinal Objectives of Secondary Education" enunciated by the Committee of the National Educational Association in 1918 or the later pronouncement by the Committee of the Department of Superintendence in 1928[3], we find nothing sufficiently definite and specific to warrant the choice of any particular subject-matter.

If these are indeed the major objectives of education, then much remains to be done to make them meaningful in terms of particular methods and specific subject-matter. The answer to this question must await further definition and experimentation.

Some Present Issues

While waiting for some more scientific answers to the above questions there appear to be some present issues which should be isolated for discussion.

Subject-matter in physics courses (or in physics texts) seems definitely to be changing. Not that we have as yet gone far in any direction from the old college preparatory physics which has been taught for the last thirty-five years. But recent texts have included newer material; they have in some instances greatly increased in size; others have changed the method of presentation. It is pertinent to ask, therefore, in what direction we are tending. Let us mention just two of the present issues.

First, have we reached a point where the old logical division into different units, as light, heat, mechanics, etc., may be profitably dispensed with, and a newer principle of organization applied, which will better coordinate the various branches of the subject?

[3]Sixth Yearbook, Department of Superintendence of the National Education Association. Chapt. II.

Second, shall we stress the subject-matter which will serve in explaining the useful appliances in daily life, or shall we add subject-matter which will attempt to bring more meaning into the larger phases of the environment?

To make the meaning of this question clear, let us quote from two of the recent texts which appear to hold the first of these as one of their major purposes. On the first page of *Practical Physics*, by Black and Davis (1926), we find:

> "On farms people mow, reap, and thresh grain with machines which are often drawn by mechanical horses called tractors; . . . In the city people travel on electric cars and go upstairs on hydraulic or electric elevators; . . . The telephone, the automobile, and the motion-picture machine, which are now familiar objects, are applications of the principles of physics.
> ". . . That is, almost everyone has begun to study *physics*, for it is one of the chief aims of physics to discover all that can be known about such machines as have just been mentioned."

On pages 1 and 2 of *Modern Physics*, by Charles E. Dull (1929), we find:

> "Men fly through the air at incredible speed; they drive racing cars at more than two hundred miles per hour; . . .
> "We turn on a switch and electricity lights our home, toasts our bread, washes our clothes or our dishes; . . .
> ". . . To select musical instruments properly or to select the correct lights and fixtures for the home, one needs a knowledge of physics. In each of the subdivisions of physics—mechanics, heat, sound, light and electricity—the questions for which we seek answers are, "What does it do?" "How does it work?", and since these answers will affect our safety and what we can afford to pay, we ask, "How much can it do?" and "What will it cost?"

Verily there is reason for the oft repeated criticism that we are machine-minded and money-mad.

Now to set off the opposite type of objectives—the interpretation of the larger phases of the environment—we can probably not do better than to refer to the type of thing attempted in two recent books which, undoubtedly, every physics teacher has read or plans to read. I refer to Eddington's "The Nature of the Physical Universe" and Jean's "The Universe Around Us," both of which have achieved a well-deserved popularity. The aim of these authors is apparently not to explain our machines, but our environment—the Universe itself.

It may not be out of place to emphasize this type of objective as another point of view from which physics may be presented. The idea of "practical" use seems to be dominant at present.

The few investigations dealing with the value of Physics to the average person, which have come to the writer's attention, have dealt with this practical aspect of the question. Too little attention, it would seem, has been paid to investigations of the ways in which physics may change one's *thinking* about the Universe and give one the thrill of a new discovery,—a new thought concept. We forget that it may actually be more important to be familiar with the latest concepts which attempt to explain the Universe,—to achieve a mental organization of the environment,—than to be able to adjust a carburetor, pick out a light fixture or tinker with a water faucet.

Obviously, in conducting scientific studies, it is far easier to analyze changes in what people do than changes in what people think and so the former type of study is bound to predominate. But we ought not let its "practical" implications dominate our texts without making an earnest endeavor to be sure that the other point of view is really of lesser importance.

We hasten to disclaim any desire to single out the two texts quoted above for specific criticism. More than one study has shown that the differences in physics texts is probably less than in texts of almost any other subject. These two are certainly as worthy as the others. Nor do we wish to contend that the present texts do not contain both types of material, nor that the up-to-date teachers are not alive to the main possibilities in both types of objective.

The issue we wish to raise is as to whether the future texts shall continue to multiply practical applications and illustrations of the principles which have been taught for the last three decades, or shall add, instead, enlightening and coordinating material on the general concepts which come out of the research of the last twenty-five years; whether new scientific studies of the values of physics shall concentrate on the "practical" uses in the humdrum of life or attempt to discover the ways in which physics may change our thinking about the universe; and at the same time devise some method of arriving at the relative importance of these two types of objectives.

Some Next Steps

The proper steps for determining rationally the proper subject-matter for the physics course would seem to be:

1. Determine as authoritatively and scientifically as possible the relative importance of those objectives which should govern the choice of subject-matter and method in physics. This should be done, of course in relation to the general principles of all education, and where possible by objective means.

2. Determine as objectively as possible the specific subject-matter materials which will realize these objectives and devise a synthesis of the subject-matter elements so obtained.

In this connection the writer is not ignorant of the splendid contributions which have already appeared in this field: the studies, for instance, which Dr. F. D. Curtis summarized in *Investigations in the Teaching of Science* (1926). We simply point out that the surface has really only been scratched; granting, of course, the splendid contributions in devising methods of attack and perfecting techniques without which no forward step would be possible.

But to make the point clear, we might suggest that of the nine subject-matter purposes listed in the first part of this paper, only two appear to have been studied objectively in any of the 44 curricular studies summarized in the book of "Investigations" named above. And even here, the purpose most frequently studied—(some nine studies on child and adult interests in science)—might be objected to as criteria for the purpose of choosing subject-matter, because of the apparent truth that people will ask questions about the things with which they come in contact, whereas a set of different contacts may elicit a whole flood of new questions. Only when provision is made to expose the experimental group to every possible type of science experience can the questions asked be considered safe criteria for judging which particular elements of science are of greatest interest to the group.

It has been the purpose of this paper to raise some questions concerning the place of subject-matter in the teaching of physics. They appear to the writer to have no obvious answers.

The statement of the questions may stimulate additional experimentation so that finally a scientific curriculum may be achieved in this scientific subject.

EXPERIMENTS IN MICROSCOPY. DEMONSTRATING BROWNIAN MOVEMENT AND COTTRELL PRECIPITATON

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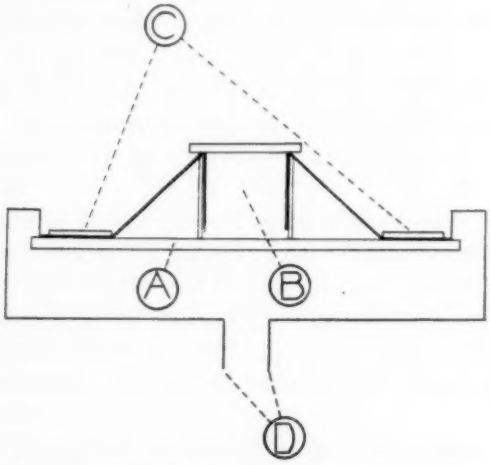
The phenomenon of the Brownian movement invariably excites keen interest when observed for the first time. As early as 1828 the English botanist Brown subjected the ceaseless, erratic motion of a suspension of pollen grains to study and experiment (1) (2) (3). It was not until 1863, however, that Wiener advanced the view that the cause of the phenomenon was the actual bombardment of the suspended particles by the molecules of the suspending medium (4) (5).

Various suspensions have been used to demonstrate the Brownian movement. Doane and Dow use arsenious sulfide (6). Black has described the preparation of a rutile suspension (7). Water suspensions of carmine and India ink are commonly utilized. Perrin used mastic and calculated Avogadro's number from his experimental data (8) (9).

In this paper the writer describes the use of an artificial smoke of finely dispersed ammonium chloride. Cigarette smoke can also be used but the phenomenon is not as striking in its effect. Using a smoke suspension for observation of the Brownian movement permits subsequent demonstration of Cottrell precipitation phenomenon on a micro scale.

A small square dish placed on a microscope stage contains the smoke. If such a dish is not available a cell may be readily constructed by fastening four square cover glasses on a microscope slide by means of ambroid cement. On opposite sides of this cell fasten long, thin strips of tinfoil to serve as electrodes. The ends of the foil are connected to thin copper wires (B. & S. Gauge No. 34) by placing the wire in contact with the foil under a cover glass and holding the latter in place by means of the stage clamps. The construction of the cell and arrangement of the electrodes are shown in the figure. A glass slide is cut

to fit the top of the cell and serves as a lid to keep in the smoke. The two copper wires are connected to the secondary of a Ford coil or other small spark coil. Stone (10) has given a good description of an apparatus to be used as a lecture demonstration of Cottrell precipitation and his article is recommended to such readers as contemplate this experiment. The spark coil is connected to a source of current and the resistance of the primary circuit is gradually increased to the point at which there is no sparking between the tinfoil electrodes. The apparatus is now ready for the experiment.



CONSTRUCTION OF GLASS CELL FOR OBSERVATION OF BROWNIAN MOVEMENT AND COTTRELL PRECIPITATION. A, IS A MICROSCOPE SLIDE; B, THE GLASS CELL; C, CONNECTION OF TINFOIL ELECTRODES AND EXTERNAL CIRCUIT; D, WIRES TO SECONDARY OF SPARK COIL. LIGHT IS PASSED THROUGH THE CELL VERTICAL TO THE PAGE.

Put a drop of concentrated ammonium hydroxide in the bottom of the cell by means of a pipet and allow to stand for several minutes. When the ammonia has vaporized drop a small square of filter paper, previously dipped in concentrated hydrochloric acid, into the cell and cover the latter with the glass lid. The cell is immediately filled with a dense cloud of ammonium chloride.

Place a strong electric light bulb behind a shade having an opening approximately $\frac{1}{8}$ inch in diameter. The pencil of light from this opening is focussed on the smoke by

means of a double-convex lens (common reading glass). Darken the room. The Brownian movement is observed by focussing a low powered objective on the beam of light passing through the smoke. The principle of the ultramicroscope (11) can be illustrated by use of this apparatus.

After the Brownian movement has been observed complete the primary circuit of the spark coil. The white fumes of ammonium chloride are immediately precipitated leaving the cell clear. The whole experiment can now be repeated by adding a drop of ammonium hydroxide and again going through the given procedure.

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(9) PERRIN, Compt. rend., 152, 1380 (1911).
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FILTERABLE GERMS OUTNUMBER VISIBLE ONES IN MILK AND SOIL

The filterable forms of germs, which are so small that they cannot ordinarily be seen even with the most powerful microscopes, far outnumber their larger, microscopically visible relatives in such substances as milk and milk products, soil, hay infusions and decomposing manure, experiments by Prof. J. M. Sherman and C. E. Safford of Cornell University show.

The ordinary forms of germs may be found by the million or even billion in one gram, which is roughly one-thirtieth of an ounce, in certain types of decomposing and fermenting materials. But the Cornell investigators found not millions but trillions of the minute organisms to the gram in some samples of milk, they reported to the journal, Science.

Contamination of milk and water with germs is ordinarily determined by a method of counting the number of microscopically visible germs or bacteria present in a given sample. But such examinations take no account of the invisible organisms which may be present. This class of invisible, filter-passing organisms is becoming increasingly important, especially since it has been found that some ordinarily visible germs may have an invisible stage. So the method developed by Prof. Sherman and Mr. Stafford may be of considerable practical significance.

The new method combines a series of dilutions of the material to be examined with a longer period for growth in broth at a temperature of about 85 degrees Fahrenheit.—*Science Service.*

COMPLEX NUMBERS IN ELEMENTARY MATHEMATICS

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A recent article written by Professor Allen A. Shaw[1] is so encouraging that I venture to suggest a few additional applications of complex numbers in elementary mathematics besides those already discussed.[2]

As Professor Shaw remarks it is unpedagogical to feed students with promises of future applications in the theory of functions, especially—I am tempted to add, when a teacher knows that most students will never get as far as the theory of functions. Complex numbers are used quite extensively in electrical engineering but only a few students and engineers are sufficiently familiar with and have sufficient confidence in the complex algebra to avail themselves to the fullest extent of the potentialities latent in this powerful method for solving problems. I believe that this undesirable situation may be remedied if the student is given an opportunity to use complex numbers in the more familiar fields such as elementary geometry, common trigonometry, analytical geometry and calculus.

In the three articles already referred to, the applications of complex numbers were based largely on their addition and subtraction but one can hardly suspect the power of complex algebra if no use is made of multiplication and division. The multiplication of complex numbers is an algebraic analogue of orthogonal and similarity transformations in Euclidean geometry. When we multiply by a complex number, geometrically speaking, we rotate a geometric figure and then transform it into a similar figure. Since the proofs of so many theorems of geometry and trigonometry are based upon the invariance of geometric figures under the group of rotations and on similarity relations and since the laws of complex algebra are formally equivalent to these, it is not surprising that simple

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[1]Shaw, Allen A., Geometric Applications of Complex Numbers, School Science and Mathematics, Vol. 31, No. 6 (1931), pp. 754-761.

[2]Smail, Lloyd L., Some Geometric Applications of Complex Numbers, American Mathematical Monthly, Vol. 36 (1929), pp. 504-511. Schelkunoff, S. A., Note on Geometrical Applications of Complex Numbers, American Mathematical Monthly, Vol. 37 (1930), pp. 301-303. John Mills, Bull. Soc. Prom. Eng. Ed., vol VIII, No. 5 (1918).

algebraic relations between complex numbers often are the equivalents of fairly complex geometric theorems. We hope that the following pages will make clear the advantages to be gained from so defining the multiplication of complex numbers that it becomes an algebraic equivalent of certain geometric transformations. One of the consequences of this change in the point of departure is that the multiplication of complex numbers in the polar form which is usually proved by recourse to the trigonometric addition theorems is established independently and the latter are derived from it. There are problems however in which the use of complex numbers is a handicap rather than an advantage; we believe the students should learn this by experience and for this reason we include some such problems in the following pages.

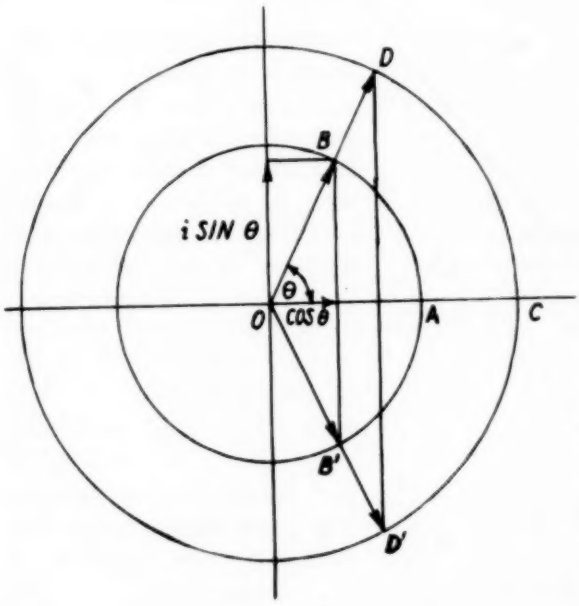

FIG. 1

After the student has been initiated into the mysteries of locating points on a straight line and in a plane by means of "coordinates" he may find it very illuminating to learn that the method is not confined to "straight" geometric objects. To locate a point on a circle (the inner circle of Fig. 1) we fix a reference point and agree on the "positive" direction. The length of arc AB, or still better, its ratio to the radius (which is the length of AB if the radius

[3]A still more natural coordinate would be the length of AB if the length of the entire circumference were chosen as the unit. Such a coordinate is intrinsic in the sense that it depends upon the curve only and not on its relations to the plane that contains it.

is chosen as the unit of length) can be taken as the coordinate of point B.[3]

Let us name this coordinate θ. We see that coordinate θ is the so-called radian measure of the $\angle AOB$. Let us now choose the cartesian coordinate system with an origin at the center as shown in Fig. 1. The cartesian coordinates of the typical point B on the unit circle are called, respectively, the cosine and the sine of θ and designated correspondingly by $\cos\theta$ and $\sin\theta$; the coordinates of point D on the circle of radius r are

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta.$$

The vectors $\overline{OB}$ and $\overline{OD}$ as well as their ends B and D are given respectively by the complex numbers $\cos\theta + i\sin\theta$ and $r\cos\theta + ir\sin\theta$. The vectors $\overline{OB'}$ and $\overline{OD'}$ are represented by the conjugate complex numbers $\cos\theta - i\sin\theta$ and $r\cos\theta - ir\sin\theta$.

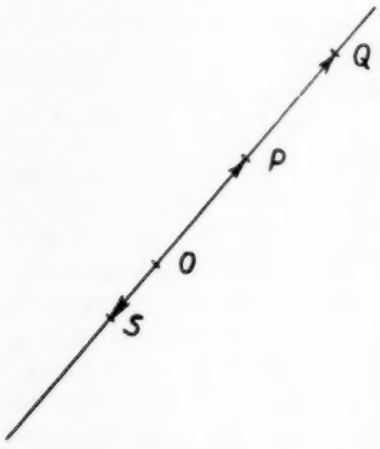

FIG. 2

After the addition and subtraction of complex numbers has been defined to conform with the addition and subtraction of vectors, the next step is to define multiplication as an operation upon vectors. We can accomplish it as follows. To multiply a vector $\overline{OP}$ (Fig. 2) by a real number r is to change its length in the ratio[4] r to 1; thus $\overline{OQ} = 2\overline{OP}$ and $\overline{OS} = -\frac{1}{2}\overline{OP}$. The distributive law is nothing but an algebraic expression of the similarity of two triangles or parallelograms (Fig. 3).

$$(1) \quad r(\overline{OA} + \overline{OB}) = r\,\overline{OA} + r\,\overline{OB}.$$

Thus we also have

$$(2) \quad a(x + iy) = ax + iay$$

if a is real.

We now define the multiplication of a vector by "i" as counterclockwise rotation of the vector through 90° (Fig. 4); thus $\overline{OQ} =$

[4] If r is positive $\overline{OP}$ and r times $\overline{OP}$ are to have the same direction; if r is negative opposite directions.

$i\ \overline{OP}$. This definition should really precede the use of such forms as $x+iy$ or $\cos\theta+i\sin\theta$. The commutative law, that is, the equivalence of such expressions as $3i$ or $i3$ is a consequence

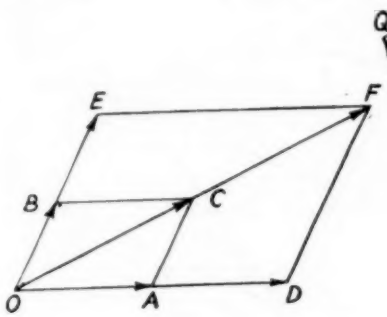

FIG. 3

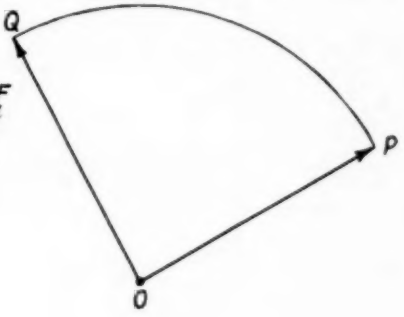

FIG. 4

of a very simple geometric relation[5] (Fig. 6). Here the distributive law is merely an assertion that if a parallelogram (Fig. 5) is rotated through 90° then its diagonal is also rotated through 90°.

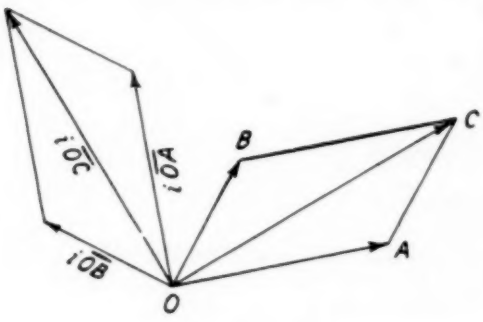

FIG. 5

(3) $i\ (\overline{OA}+\overline{OB}) = i\ \overline{OA}+i\ \overline{OB}$.

Thus we have

(4) $i^2=-1, i^3=-i, i^4=1, i^5=i$, etc.

The multiplication of a vector by a complex number $\cos\theta+i\sin\theta$ is defined to conform with the distributive law

(5) $(\cos\theta+i\sin\theta)\ \overline{OA}=(\cos\theta)\ \overline{OA}+(i\sin\theta)\ \overline{OA}$. Thus (Fig. 7), we have $\overline{OM}=(\cos\theta)\ \overline{OA}$, $\overline{ON}=(i\sin\theta)\ \overline{OA}$ and the effect of multiplication is to rotate the vector $\overline{OA}$ through angle θ.

From the above discussion we conclude that the theory of complex numbers can be founded upon the geometric definitions. This theory could be made as rigorous as geometry itself but for our present purposes it is better to pass over the details of proof and consider a number of applications.

[5] I.e., the final result is the same whether we rotate a vector through 90° and then treble it, or we treble it first and then rotate through 90°.

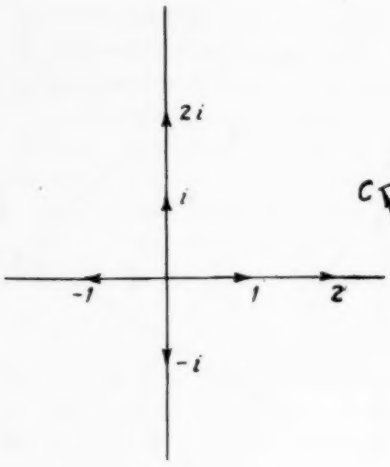

FIG. 6

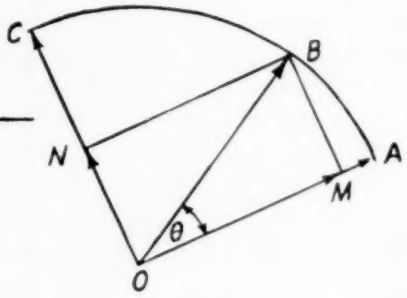

FIG. 7

We shall begin with the derivation of some trigonometric identities. Thus if

$$z_1 = \cos\theta + i\sin\theta$$

and

$$z_2 = iz_1$$

we have on the one hand directly

$$z_2 = \cos(\theta+\frac{\pi}{2}) + i\sin(\theta+\frac{\pi}{2})$$

because the vector z_1 is rotated through $\frac{\pi}{2}$, and on the other hand

$$z_2 = i(\cos\theta + i\sin\theta) = i\cos\theta - \sin\theta.$$

Therefore

$$\cos(\theta+\frac{\pi}{2}) = -\sin\theta,\ \sin(\theta+\frac{\pi}{2}) = \cos\theta.$$

Similarly

$$i^2(\cos\theta + i\sin\theta) = \cos(\theta+\pi) + i\sin(\theta+\pi)$$

and also

$$i^2(\cos\theta + i\sin\theta) = -(\cos\theta + i\sin\theta) = -\cos\theta - i\sin\theta$$

so that

$$\cos(\theta+\pi) = -\cos\theta,\ \sin(\theta+\pi) = -\sin\theta.$$

Likewise, we can prove

$$\cos(\theta+\frac{3\pi}{2}) = \sin\theta,\ \sin(\theta+\frac{3\pi}{2}) = -\cos\theta.$$

The more general addition theorems are just as easy to establish. Thus if

$$z_2 = (\cos\theta + i\sin\theta)\, z_1$$

where

$$z_1 = \cos\varphi + i\sin\varphi$$

we have on the one hand directly (Fig. 8),

$$z_2 = \cos(\theta+\varphi) + i\sin(\theta+\varphi),$$

and on the other hand

$$\begin{aligned} z_2 &= (\cos\theta + i\sin\theta)(\cos\varphi + i\sin\varphi) \\ &= \cos\theta(\cos\varphi + i\sin\varphi) + i\sin\theta(\cos\varphi + i\sin\varphi) \\ &= (\cos\theta\cos\varphi - \sin\theta\sin\varphi) + i(\sin\theta\cos\varphi + \cos\theta\sin\varphi). \end{aligned}$$

Equating the real and imaginary parts of z_2 as given in the two forms, we have

$$(6)\quad \begin{aligned} \cos(\theta+\varphi) &= \cos\theta\cos\varphi - \sin\theta\sin\varphi \\ \sin(\theta+\varphi) &= \sin\theta\cos\varphi + \cos\theta\sin\varphi \end{aligned}$$

without any restriction imposed on the size of the angles θ and φ.

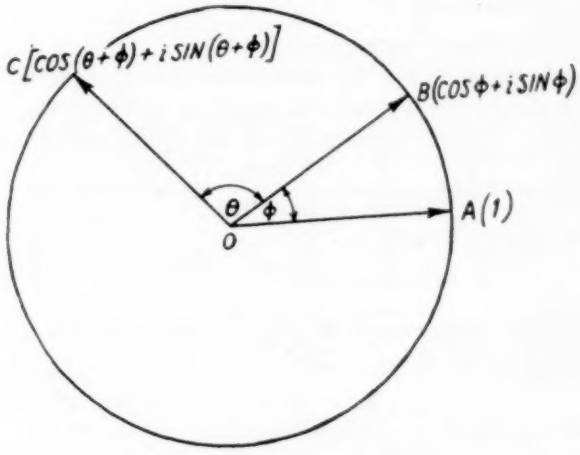

FIG 8

In contradistinction to the customary proofs of addition theorems based on direct recourse to certain more or less special geometric theorems, we have here introduced some important geometric postulates in the definition of multiplication of complex numbers. Hence we have the simplicity and greater generality of proofs based on the complex algebra but we have attained this simplicity by sacrificing the purity of geometric methods. This is hardly an objection, however, since trigonometry is after all closer to algebra than to geometry.

The extended addition theorems can be obtained with just as little effort thus

$$(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)(\cos\theta_3+i\sin\theta_3)$$
$$=\cos(\theta_1+\theta_2+\theta_3)+i\sin(\theta_1+\theta_2+\theta_3)=\cos\theta_1\cos\theta_2\cos\theta_3$$
$$-\cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_1\sin\theta_2-\cos\theta_3\sin\theta_1\sin\theta_2$$
$$+i(\sin\theta_1\cos\theta_2\cos\theta_3+\sin\theta_2\cos\theta_1\cos\theta_2+\sin\theta_2\cos\theta_1\cos\theta_2$$
$$-\sin\theta_1\sin\theta_2\sin\theta_3)$$

and therefore

$$\cos(\theta_1+\theta_2+\theta_3)=\cos\theta_1\cos\theta_2\cos\theta_3-\cos\theta_1\sin\theta_2\sin\theta_3$$
$$-\cos\theta_2\sin\theta_1\sin\theta_2-\cos\theta_3\sin\theta_1\sin\theta_2;$$
$$\sin(\theta_1+\theta_2+\theta_3)=\sin\theta_1\cos\theta_2\cos\theta_3+\sin\theta_2\cos\theta_1\cos\theta_2$$
$$+\sin\theta_3\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2\sin\theta_3.$$

The law of formations of various products in the generalized addition theorems and the signs to be attached to them can be seen immediately once complex multiplication is understood.

DeMoivre's theorem

$$(7)\qquad (\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$$

is obtained immediately by repeating the process of multiplication n times which results in rotation of the vector through the angle $n\theta$. On the other hand by the binominal theorem

$$(\cos\theta+i\sin\theta)^n=\left[\cos^n\theta-\frac{n(n-1)}{1.2}\cos^{n-2}\theta\sin^2\theta+\right.$$
$$\left.+\frac{n(n-1)(n-2)(n-3)}{1\cdot2\cdot3\cdot4\cdot}\cos^{n-4}\theta\sin^4\theta-\ldots\right]+$$
$$+i\left[n\cos^{n-1}\theta\sin\theta-\frac{n(n-1)(n-2)}{1\cdot2\cdot3}\cos^{n-3}\theta\sin^3\theta+\ldots\right]$$

and therefore

$$\cos n\theta=\cos^n\theta-\frac{n(n-1)}{1\cdot2}\cos^{n-2}\theta\sin^2\theta+\ldots$$

$$\sin n\theta=n\cos^{n-1}\theta\sin\theta-\frac{n(n-1)(n-2)}{1\cdot2\cdot3}\cos^{n-3}\theta\sin^3\theta+\ldots$$

In the exercise for students n should perhaps be given special values 2, 3, 4 and so on.

Another series of interesting as well as useful trigonometric identities can be obtained as follows. Let

$$z=\cos\theta+i\sin\theta.$$

Then

$$\frac{1}{z}=\cos\theta-i\sin\theta.$$

Thus

$$\cos\theta=\frac{1}{2}(z+\frac{1}{z}). \tag{8}$$

$$\sin\theta=\frac{1}{2i}(z-\frac{1}{z})=-\frac{i}{2}(z-\frac{1}{z}). \tag{9}$$

Also

$$z^n=(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$$
$$z^{-n}=(\cos\theta-i\sin\theta)^n=\cos n\theta+i\sin n\theta$$

and therefore

$$\cos n\theta=\frac{1}{2}(z^n+\frac{1}{z^n})$$

(10) and

$$\sin n\theta=-\frac{i}{2}(z^n-\frac{1}{z^n}).$$

If we now take the n^{th} power of equation (8) we have

$$\cos^n\theta=\frac{1}{2^n}(z+\frac{1}{z})^n=\frac{1}{2^n}\Big[z^n+nz^{n-2}+$$
$$+\frac{n(n-1)}{1\cdot 2}z^{n-4}+\ \ldots\ +\frac{n(n-1)}{1\cdot 2}\frac{1}{z^{n-4}}+\frac{n}{z^{n-2}}+\frac{1}{z^n}\Big].$$

Combining the terms equidistant from the ends of the expansion and substituting from equation (10) we have

$$\cos^n\theta=\frac{1}{2^{n-1}}\Big[\cos n\theta+n\cos(n-2)\theta+ \tag{11}$$
$$\frac{n(n-1)}{1\cdot 2}\cos(n-4)\theta+\ \ldots\ \Big].$$

When n is even the last term in this expansion exhibits a peculiar behavior and it may be well for the student to repeat the above derivation for special values of n.

We shall now derive the "Factorization Formulae" referring to the unit circle shown on Figure 9. Let $\angle BOA=\alpha$, $\angle COA=\beta$. Draw OD, the bisector of $\angle COB$; then $\angle COD=\frac{\beta-\alpha}{2}$ and $\angle DOA=\frac{\beta+\alpha}{2}$. The

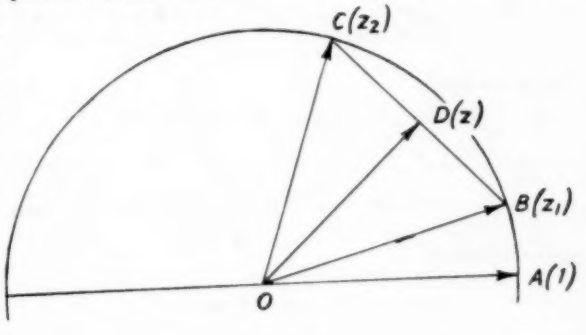

FIG. 9

point D(z) is the mid-point of BC. Therefore

$$\frac{z_1+z_2}{2}=z \quad \text{or} \quad z_1+z_2=2z.$$

But

$$z_1=\cos\alpha+i\sin\alpha,$$
$$z_2=\cos\beta+i\sin\beta$$

and since the length of $\overline{OD}$ is $\cos\frac{\beta-\alpha}{2}$ we have also

$$z=\cos\frac{\beta-\alpha}{2}\left[\cos\frac{\beta+\alpha}{2}+i\sin\frac{\beta+\alpha}{2}\right].$$

Therefore

$$(\cos\alpha+\cos\beta)+i(\sin\alpha+\sin\beta)=$$
$$=2\cos\frac{\beta-\alpha}{2}\cos\frac{\beta+\alpha}{2}+2i\sin\frac{\beta+\alpha}{2}\cos\frac{\beta-\alpha}{2}.$$

Equating the real and imaginary parts

$$\cos\alpha+\cos\beta=2\cos\frac{\beta-\alpha}{2}\cos\frac{\beta+\alpha}{2}.$$

(12)

$$\sin\alpha+\sin\beta=2\cos\frac{\beta-\alpha}{2}\sin\frac{\beta+\alpha}{2}.$$

We also have

$$\overline{DC}=\frac{z_2-z_1}{2}.$$

But the length of $\overline{DC}$ is $\sin\frac{\beta-\alpha}{2}$, the vector being perpendicular to the $\cos\frac{\beta+\alpha}{2}+i\sin\frac{\beta+\alpha}{2}$ so that

$$\overline{DC}=i\sin\frac{\beta-\alpha}{2}\left[\cos\frac{\beta+\alpha}{2}+i\sin\frac{\beta+\alpha}{2}\right].$$

Thus we have

$$\frac{\cos\beta-\cos\alpha}{2}+i\,\frac{\sin\beta-\sin\alpha}{2}$$
$$=i\sin\frac{\beta-\alpha}{2}\cos\frac{\beta+\alpha}{2}-\sin\frac{\beta-\alpha}{2}\sin\frac{\beta+\alpha}{2}.$$

and therefore

$$\cos\beta-\cos\alpha=-2\sin\frac{\beta-\alpha}{2}\sin\frac{\beta+\alpha}{2}$$

(13)

$$\sin\beta-\sin\alpha=2\sin\frac{\beta-\alpha}{2}\cos\frac{\beta+\alpha}{2}.$$

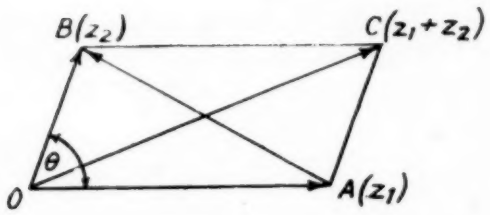

FIG. 10

Let us now calculate the diagonals of a parallelogram (Fig. 10) whose sides a and b and angle θ between them are known. Let us now choose the real axis along $\overline{OA}$.

Then $z_1=a$, $z_2=b(\cos\theta+i\sin\theta)$ and therefore

$$z_1+z_2=(a+b\cos\theta)+i\,b\sin\theta$$
$$z_2-z_1=(b\cos\theta-a)+i\,b\sin\theta.$$

Taking the "norms" (the squares of the absolute values of these complex numbers we have

$$|z_1+z_2|^2=OC^2=(a+b\cos\theta)^2+b^2\sin^2\theta=a^2+2ab\cos\theta+b^2,$$
$$|z_1-z_2|^2=AB^2=(b\cos\theta-a)^2+b^2\sin^2\theta=a^2-2ab\cos\theta+b^2. \tag{14}$$

In trigonometry the latter equation is known as the "Theorem of Cosines."

The classical relation between binominal equations in algebra and division of circles into equal parts in geometry is so simple that it is a pity that many college students never even hear of it. Referring to Figure 11 we see that the division of the circle into six equal parts and the solution of equation

$$z^6=1 \quad \text{or} \quad z^6-1=0$$

are related problems. And in general to divide a circle into "n" equal parts is to solve $z^n=1$ or $z^n-1=0$ and vice versa.

Some interesting and useful identities can be obtained from Figure 11. Thus by symmetry we conclude that $1+z+z^2+\ldots z^5=0$. If more formal proof is desired, we note that the sum on the left is

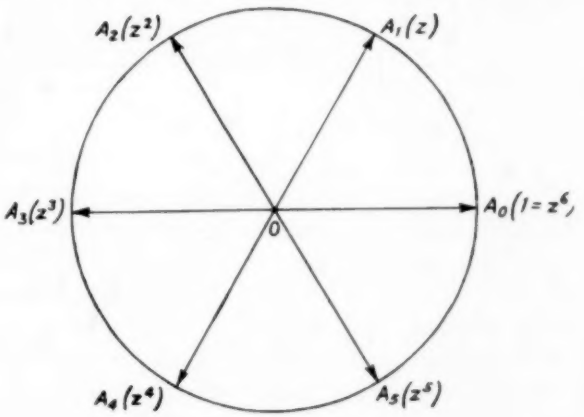

FIG. 11

the sum of the roots of the equation $z^6 - 1 = 0$ and is 0 because the coefficient of z^5 is 0. Or else we can factor the preceding equation and write $(z-1)\ (1+z+z^2+ \ . \ . \ . \ +z^5)=0$ and if z does not equal 1, $1+z+z^2+ \ . \ . \ . \ +z^5=0$.

Thus we have

$$1+(\cos\ \theta+i\ \sin\ \theta)+(\cos\ 2\theta+i\ \sin\ 2\theta)+ \ . \ . \ . \ +(\cos\ 5\theta+i\ \sin\ 5\theta)=0$$

where

$$\theta=\frac{2\pi}{6}.$$

Equating the real and imaginary parts to 0 we have

$$1+\cos\frac{2\pi}{6}+\cos\frac{4\pi}{6}+ \ . \ . \ . \ +\cos\frac{10\pi}{6}=0$$

$$\sin\frac{2\pi}{6}+\sin\frac{4\pi}{6}+ \ . \ . \ . \ +\sin\frac{10\pi}{6}=0.$$

In general we can prove

$$1+\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+ \ . \ . \ . \ +\cos\frac{2(n-1)\pi}{n}=0$$

$$\sin\frac{2\pi}{n}+\sin\frac{4\pi}{n}+ \ . \ . \ . \ +\sin\frac{2(n-1)\pi}{n}=0.$$

To obtain a more general identity we refer to Figure 12 and write

$$[\cos\ \theta+i\ \sin\ \theta]$$
$$+\left[\cos(\theta+\frac{2\pi}{n})\right.$$
$$\left.+i\ \sin(\theta+\frac{2\pi}{n})\right]$$
$$+\left[\cos(\theta+\frac{4\pi}{n})\right.$$
$$\left.+i\ \sin(\theta+\frac{4\pi}{n})\right]+ \ . \ . \ .$$
$$+\left[\cos(\theta+\frac{2(n-1)\pi}{n})\right.$$
$$\left.+i\ \sin(\theta+\frac{2(n-1)\pi}{n})\right]=0$$

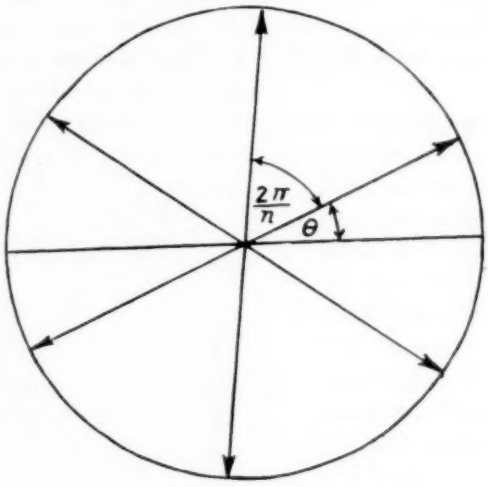

FIG 12

from which the referred to identities are obtained by equating the real and imaginary parts to 0.

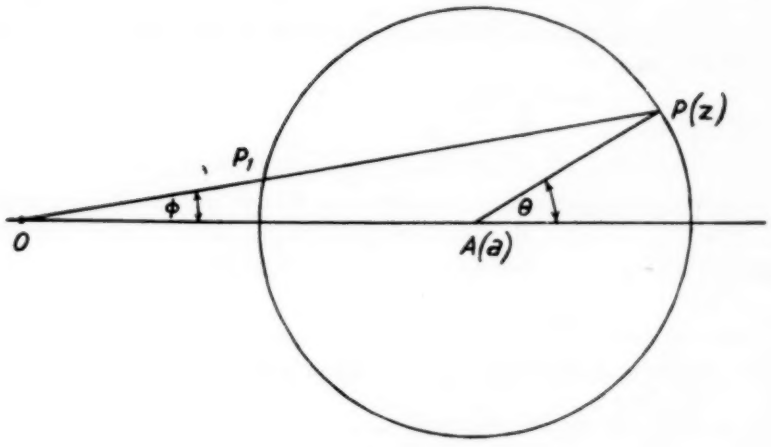

FIG. 13

Let us now prove a theorem concerning the segments of a secant drawn to a circle from a fixed point on Figure 13. Choosing OA as the real axis we find that a typical point P on the circle is given by the equation $z_1=a+r(\cos\theta+i\sin\theta)$ where r is the radius and θ is a variable parameter. A typical point on the straight line OP is given by $z_2=t(\cos\varphi+i\sin\varphi)$ where t is a real variable parameter. For the points of intersection of the straight line and the circle we have $z_1=z_2$, i. e., $a+r(\cos\theta+i\sin\theta)=t(\cos\varphi+i\sin\varphi)$. Let us now look for a relation *independent* of θ; i. e., a relation true, irrespective of the position of the point P on the circle. For this purpose let us transfer a to the other side of the above equation; thus we have $r(\cos\theta+i\sin\theta)=(t\cos\varphi-a)+it\sin\varphi$. Since the complex numbers are equal their norms are also equal; thus

$$r^2=t^2-2at\cos\varphi+a^2$$

or

$$t^2-2at\cos\varphi+(a^2-r^2)=0.$$

This is a quadratic equation showing that for each inclination φ less than a certain limiting value we have two points P corresponding to two values of t. The product of these roots is

$$t_1t_2=a^2-r^2$$

and therefore independent of φ. Thus, *the product of the segments OP_1 and OP of two secants drawn to a circle through a fixed point O depends only upon the radius of the circle and the position of the fixed point. This product equals the square of the tangent drawn from the point O to the circle.*

Since by definition t is real and the roots of the above quadratic are real only if

$$a^2\cos^2\varphi \geq a^2 - r^2$$

or

$$r^2 \geq a^2 - a^2\cos^2\varphi$$

$$\sin\varphi \leq \frac{r}{a};$$

the limiting angle φ_0 is determined by the equation

$$\sin\varphi_0 = \frac{r}{a}$$

If $\varphi > \varphi_0$ the secant does not intersect the circle.

It can hardly be doubted that some students will wonder if areas and work are expressible by means of complex numbers. The answer is yes, of course; the method, however does not seem to offer any advantages in elementary applications, although I have found it very convenient in many practical applications of the classical electromagnetic theory.

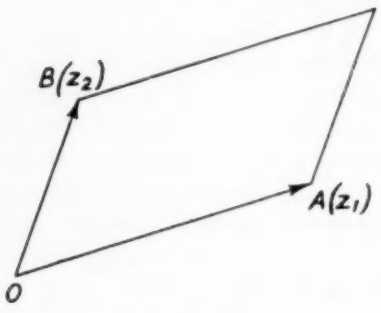

FIG. 14

Suppose (Fig. 14) we have two vectors z_1 and z_2 and $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$. Then if z^* is the conjugate of z, we have

$$z_1^* z_2 = r_1 r_2\cos(\theta_2 - \theta_1) + i\, r_1 r_2 \sin(\theta_2 - \theta_1).$$

Thus if the vectors z_1 and z_2 are interpreted respectively as a force acting upon a particle and a displacement of a particle, then the real part of $z_1^* z_2$ is the work done by the force; if z_1 and z_2 are merely the sides of a parallelogram, then the "i-coefficient" of $z_1^* z_2$ is the area of the parallelogram. In the latter case the area $\triangle$ is

$$\triangle = i\text{-coefficient of } z_1^* z_2 = \frac{z_1^* z_2 - z_1 z_2^*}{2i}.$$

If the diagonals of the parallelogram are

$$w_1 = z_1 + z_2 \quad \text{and} \quad w_2 = z_1 - z_2$$

and therefore

$$z_1 = \frac{w_1 + w_2}{2} \quad \text{and} \quad z_2 = \frac{w_1 - w_2}{2},$$

we can express the area of the parallelogram in terms of the diagonals

$$\Delta = i\text{-coefficient of } \frac{w_1^*+w_2^*}{2}\cdot\frac{w_1-w_2}{2}$$

$$= i\text{-coefficient of } \tfrac{1}{4}[w_1^*w_1 - w_2w_2^* + w_1w_2^* - w_1^*w_2].$$

Since the first two terms are real and the difference between the last two is purely imaginary, we have

$$\Delta = \frac{w_1w_2^* - w_1^*w_2}{4i}.$$

The area of a parallelogram whose sides are respectively equal and parallel to the diagonals of the given parallelogram is

$$A = \frac{w_1w_2^* - w_1^*w_2}{2i}$$

and thus is seen to be twice the area of the original parallelogram. This theorem admits other far simpler proofs; no method, however, is the best on every occasion and the student might just as well be aware of the relative advantages and disadvantages of any particular method.

The equation of a straight line, that is the equation satisfied by the complex number defining the typical point on the straight line can be determined as follows. Let the straight line in question pass through two fixed points $A(z_1)$ and $B(z_2)$ (Fig. 15). Let a typical point on the line be $P(z)$. Then the phases of the complex numbers $z-z_1$ and z_2-z_1 are either equal or differ by π. In any case the ratio of the complex numbers is real, t let us say.

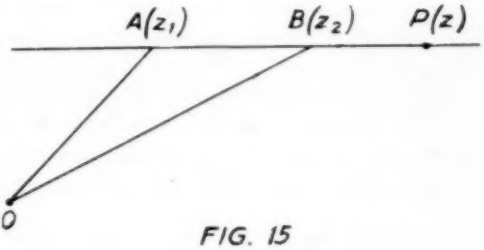

FIG. 15

Thus

$$z-z_1 = t(z_2-z_1) \quad \text{or} \quad z = z_1 + t(z_2-z_1).$$

This is of the form

$$z = at+b$$

where a and b are complex constants and t is a *real* variable parameter. A straight line passes through the origin if

$$at+b=0 \quad \text{or} \quad t=-\frac{b}{a},$$

that is, if the ratio of b to a is real. By changing the variable

parameter, the equation of such a line can always be written as follows

$$z = at.$$

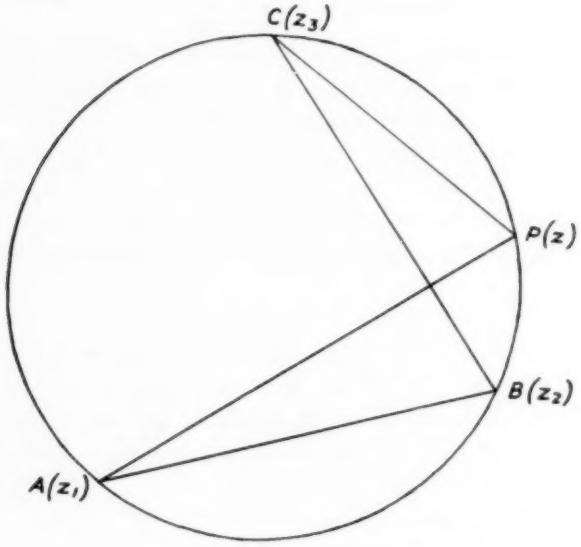

FIG. 16

Let us now determine the complex number corresponding to the typical point P on the circle passing through three different points $A(z_1)$, $B(z_2)$ and $C(z_3)$ (Fig. 16). Since the angles ABC and APC are either equal or supplementary, the phases of $\frac{z_1-z_2}{z_3-z_2}$ and $\frac{z_1-z}{z_3-z}$ are, respectively, either equal or differ by π. Therefore the ratio of these complex numbers is a real number t, let us say. Thus

$$\frac{(z_1-z)\ (z_3-z_2)}{(z_3-z)\ (z_1-z_2)} = t.$$

Solving this for z, we have

$$z = \frac{tz_3(z_1-z_2)-z_1(z_3-z_2)}{t(z_1-z_2)-(z_3-z_2)}$$

where t is a real parameter. The points $A(z_1)$, $B(z_2)$ and $C(z_3)$ correspond respectively to $t=0,\ 1,\ \infty$. Thus the equation of the circle can be written in the following convenient form

$$z = \frac{at+b}{ct+d}$$

where a, b, c and d are complex constants and t is a real vari-

able parameter.

Let

$$w=\frac{kt+l}{mt+n}$$

be the equation of another circle. Taking the parameter t the same in the equations of both circles, we establish a 1–1 correspondence between the points of the latter. Solving one of these equations for t and substituting into the other, we have

$$w=\frac{\alpha z+\beta}{\gamma z+\delta}$$

where α, β, γ, and δ are some expressions depending upon a, b, c, d, k, l, m and n. Thus, if two complex variables are related as above and one of these describes a circle the other also describes a circle. However, this takes us perhaps beyond the field of elementary mathematics.

Heretofore we have dealt with applications of complex numbers in geometry and trigonometry and it befits our purpose to take two or three examples from the field of analysis. There is no reason why a student of calculus should not become acquainted with Fourier series. Fourier expansions of certain functions can be obtained by strictly elementary means; thus as soon as we derive an expansion of some function into a power series we can immediately obtain two Fourier expansions. For instance starting with

$$\frac{1}{1-z}=1+z+z^2+z^3+\ \ldots$$

and letting

$$z=r\ (\cos\ \theta+i\ \sin\ \theta)$$

we have

$$\frac{1}{(1-r\ \cos\theta)\ -ir\ \sin\ \theta}=1+r\ (\cos\ \theta+i\sin\ \theta)$$
$$+r^2\ (\cos\ 2\theta+i\ \sin\ 2\theta)+\ \ldots$$

But the left hand side can be reduced to

$$\frac{(1-r\ \cos\ \theta)+ir\ \sin\ \theta}{1-2r\ \cos\ \theta\ +r^2},$$

so that

$$\frac{1-r\ \cos\ \theta}{1-2r\ \cos\ \theta+r^2}=1+r\ \cos\ \theta+r^2\ \cos\ 2\theta+\ \ldots \tag{15}$$

$$\frac{r\ \sin\ \theta}{1-2r\ \cos\ \theta+r^2}=r\ \sin\ \theta+r^2\ \sin\ 2\theta+\ \ldots \tag{16}$$

As another example we can take

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots,$$

and put as before

$$z = r(\cos\theta + i\sin\theta).$$

Thus we have

$$e^{r\cos\theta + ir\sin\theta} = e^{r\cos\theta}[\cos(r\sin\theta) + i\sin(r\sin\theta)]$$

$$= 1 + r(\cos\theta + i\sin\theta) + \frac{r^2}{2!}(\cos 2\theta + i\sin 2\theta) + \ldots$$

and therefore

$$e^{r\cos\theta}\cos(r\sin\theta) = 1 + r\cos\theta + \frac{r^2}{2!}\cos 2\theta + \ldots$$

$$e^{r\cos\theta}\sin(r\sin\theta) = r\sin\theta + \frac{r^2}{2!}\sin 2\theta + \ldots$$

Sometimes a slight manipulation of an established expansion yields another. Thus subtracting ½ from both sides of equation (15) we have

$$\frac{1 - \frac{1}{2}(1+r^2)}{(1+r^2) - 2r\cos\theta} = \frac{1}{2} + r\cos\theta + r^2\cos 2\theta + \ldots$$

Dividing the numerator and the denominator on the left by $(1+r^2)$ and letting

$$\frac{2r}{1+r^2} = k \tag{17}$$

we have

$$\left[\frac{1}{1+r^2} - \frac{1}{2}\right]\frac{1}{1 - k\cos\theta} = \frac{1}{2} + r\cos\theta + r^2\cos 2\theta + \ldots$$

If k is given, r is determined from equation (17) or

$$r^2 - \frac{2}{k}r + 1 = 0.$$

Thus

$$r = \frac{1 \pm \sqrt{1-k^2}}{k} = \frac{k}{1 \mp \sqrt{1-k^2}};$$

i e.,

$$r = \frac{k}{1 + \sqrt{1-k^2}},$$

since $r < 1$ by definition, and therefore the negative sign in the above expression gives an extraneous root. Also we have

$$\frac{1}{1+r^2} - \frac{1}{2} = \frac{k}{2r} - \frac{1}{2} = \frac{k-r}{2r} = \frac{\sqrt{1-k^2}}{2}$$

and therefore

$$\frac{1}{1-k\cos\theta}=\frac{1}{\sqrt{1-k^2}}\left[1+2\sum_{n=1}^{\infty}r^n\cos n\theta\right].$$

We can reverse this process and sum up certain finite and infinite series. One example will be sufficient to prove the essential elementary character of this method. Suppose we wish to find the sum of the following finite series

$$A=1+\cos x+\cos 2x+\ldots+\cos nx.$$

We observe that this is the real part of

$$S=1+z+z^2+\ldots+z^n$$

where

$$z=\cos x+i\sin x.$$

But this is a geometric progression and therefore

$$S=\frac{z^{n+1}-1}{z-1}=\frac{z^{\frac{n+1}{2}}\left(z^{\frac{n+1}{2}}-z^{-\frac{n+1}{2}}\right)}{z^{\frac{1}{2}}\left(z^{\frac{1}{2}}-z^{-\frac{1}{2}}\right)}$$

$$=\frac{\sin\frac{n+1}{2}x}{\sin\frac{x}{2}}\left(\cos\frac{nx}{2}+i\sin\frac{nx}{2}\right).$$

Hence

$$A=\frac{\sin\frac{n+1}{2}x}{\sin\frac{x}{2}}\cos\frac{nx}{2}$$

and also

$$\sin x+\sin 2x+\ldots+\sin nx=\frac{\sin\frac{n+1}{2}x}{\sin\frac{x}{2}}\sin\frac{nx}{2}.$$

In these transformations we could also have availed ourselves of Euler's identity $e^{ix}=\cos x+i\sin x$ if this were previously established.

In conclusion we express a hope that these elementary applications of complex numbers will help to make college mathematics more interesting and useful.

Federal Office of Education facts show that the average American boy or girl of 1931 receives two more years of schooling than the average boy or girl of 1914; that he is one of a class of 30 pupils, while his father's 1910 class had 34 pupils; that his chances of going to high school, which were but 1 in 10 in 1900 are now fifty-fifty, and his chances of going to college are 1 in 6.

MAPS AND GRAPHS AS TOOLS IN TEACHING GEOGRAPHY

By Edna E. Eisen,
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Maps and geography cannot be separated. To some the two words are synonymous, but this is a misconception. Maps are only one of the "tools" which pupils should be able to use to help them to gain a better understanding of relationships between human activities and natural environmental factors. It is only if and *when* they function in helping one to reach a geographic understanding, that is, an understanding of relationships of man to natural environment, that maps should be used. This is the most essential principle for us to bear in mind continually in presenting materials to our pupils and in directing their activities toward the attainment of worthwhile geographic understandings.

Terms and Symbols

Pupils need a knowledge of map language. This is not merely a knowledge of the names of symbols, or of things for which they stand, but the *ability* to *translate* the names and symbols into imagery so that they have real meaning and can function properly in geographic study. Terms and symbols which should have such vital functioning meaning to pupils when they have completed elementary geography are given in List I. Some of the items listed are "conditions," some "features." Such ideas as upstream, downstream, and direction are represented on maps, but not by specific symbols; while cities, rivers, coasts, and the like are definite features for each of which there are definite symbols.

Experience has shown that, at present, many pupils of junior and senior high schools are not equipped with this functioning meaning of these terms and symbols. A rather striking evidence of this inability may be seen in the results of a little exercise given to 192 pupils of the seventh and eighth grades. The pupils were asked to number from one to four on their papers. An outline was drawn on the board to represent an imaginary island on which was marked a stream flowing south. The symbol for a place called A was put on the stream near its source and the symbol for B near its mouth. The pupils were asked, "In traveling on

this stream from A to B can you drift or must you row?" They were told to write "drift" or "row" after number one on their papers. Similarly streams flowing east, north, and west with places A and B similarly placed, were drawn on the imaginary island and the pupils asked to answer the same question in regard to each stream, writing their answers in the appropriate place each time. Of the 192 pupils tested, 34 or 18 per cent of the group were correct in four cases; 30 per cent missed number one, 50 per cent number two, 66 per cent number three, and 33 per cent number four. These figures indicate a rather general lack of understanding of the "conditions" upstream and downstream by this group. Pupils handicapped in this way would have trouble, for example, in understanding maps showing upstream and downstream traffic on the Rhine River used in connection with a geographic unit of the industrial region of western Europe.

It is difficult for us to realize that our pupils do not visualize and sense many of the things symbolized on maps we use. To illustrate, after a study of western United States in the seventh grade had been completed, a test was given in which there were exercises to check understandings of relationships between uses of land in the various farming districts and factors of the natural environment, especially rainfall and length of growing season. In studying the errors it was discovered that over 40 per cent of the total number were in those questions concerning relationships in the Imperial Valley and on the Columbia Plateau. In reteaching the farming part of the unit, using a physical-political wall map of the United States, a map of average annual precipitation found in the pupils' texts, and crop distribution maps, the cause of the trouble was found. The pupils had been reading the dark green on the map as indicating heavy rainfall and the yellow as indicating very scant rainfall. It was this confused idea which was at the bottom of their difficulties in understanding the geographic relationships. When this was cleared up and the pupils realized that the color symbols referred only to elevations there was no more difficulty. In trying to find a reason for this misconception of ideas it was found that these pupils had been in the habit of using a physical map and jumping

at such conclusions as this: "The area is colored dark green on the map, therefore farming is important; there is, of course, much rain and fertile soil."

LIST I

Terms and Symbols

1. Map
2. Symbol or sign
3. Legend or key
4. Title
5. Sea, gulf, bay
6. Mouth and source
7. Upstream and downstream
8. River
9. Delta
10. Coast line
11. Harbor
12. Directions
13. Scale of miles
14. Strait
15. Isthmus
16. Peninsula
17. Island
18. City
19. Capital
20. Country
21. Boundary
22. Mountain—system, peak
23. Plateau
24. Passes
25. Lake
26. Valley
27. Plain
28. Railroad
29. Oceans
30. Continents
31. Up and down
32. North and South Poles
33. Equator
34. Tropic of Cancer
35. Tropic of Capricorn
36. Arctic Circle
37. Antarctic Circle
38. Degrees of latitude
39. Parallels of latitude
40. Prime meridian
41. Meridians of longitude
42. Degrees of longitude
43. Distance on a map
    a. In miles through use of scale of miles
    b. In degrees of latitude
44. Symbols for various items for which there is no standard symbol but which are explained in legends. Such items are:
    a. Average temperature
    b. Average rainfall or precipitation
    c. Population per square mile
    d. Growing season
    e. Acreage
    f. Production
45. Distribution of such items as those mentioned in 44

LIST II

Various kinds of maps showing natural phenomena

1. Distribution of population
2. Rice acreage
3. Distribution of number of cattle
4. Distribution of number of goats
5. Distribution of number of buffaloes
6. Tobacco acreage
7. Tea acreage
8. Cotton acreage
9. Flax acreage
10. Corn acreage
11. Grain sorghum and millet acreage
12. Wheat acreage
13. Barley acreage
14. Sugar-cane acreage
15. Physical map, showing also important cities and railroads
16. Maps of average summer and winter rainfall

LIST III

Various kinds of maps showing human items

1. Distribution of population
2. Distribution of crop acreage or production
3. Distribution of sheep, cattle, goats, etc.
4. Distribution of cities
5. Distribution of mineral production
6. Rank in production
7. Railroad net maps
8. Steamship routes
9. City land utilization maps
10. Distribution of other human items—irrigation projects, National Forests and National Parks, Manufacturing Areas, etc.
11. Isopleth maps of ratios
12. Good maps of physical features plus cities, railroads, political boundaries
13. Traffic maps such as tonnage and value of commodities shipped on the Great Lakes, for example

LIST IV

Various kinds of maps showing natural phenomena

1. Physical maps
2. Maps showing lengths of growing seasons
3. Maps of distribution of precipitation (summer and winter)
4. Soil maps
5. Maps of distribution of mineral resources
6. Maps of distribution of forests
7. Maps of native vegetation

Note: There has been no attempt to make these lists complete. They are given as suggestive lists of maps quite generally available either in wall maps, texts, or government publications.

Recently one of the brighter boys in a seventh grade class studying Latin America asked, "What does it mean on those maps when it says 30 inches rainfall? Is that every time it rains or what?" We were using seasonal rainfall maps to show the relation between cacao production and conditions of rainfall in Northeast Brazil. Although it would seem that the children must have used such maps frequently in earlier stages there were very few in the class who offered to give some sort of an explanation. The majority of the class read the titles "Average Precipitation (Nov. 1-April 30) and (May 1-Oct. 30)" and the legend without translating the terms into imagery. It was necessary to stop in order that the use of these maps could function in gaining the geographic understanding and explain how we might measure the rain that fell the next time we had a shower here and how in a similar fashion that is done at weather bureaus and records kept for years to determine averages. In the course of the explanation, some pupil asked about how much rain did fall each time we had a shower. If you have never heard the answers to such a question, be sure

to ask your pupils so you may see for yourself the wide variation of ideas. Another pupil asked about the average annual precipitation at home and with this fact to compare with the rainfall of the part of Brazil under discussion the difficulty seemed to be cleared and the pupils were ready to go on.

One might continue for an hour or more to relate similar experiences which we all have had. I wish to speak of only one more and that relative to items 32 to 39 in List I. I do not suppose that there is any single one map ability which is more essential than a functioning understanding of latitude. Latitude facts translate to time of season, length of season, noon position of the sun, and length of day. If these concepts of sun behavior are built up in relation to the activities of people living at different distances from the equator to the pole as a thread in a gradual, cumulative process there is little doubt but that they will function properly if brought into use when needed. But where latitude facts are treated abstractly and in connection with longitude they seldom are understood. I have used a simple sun-behavior test with hundreds of junior high school pupils who have had the usual type of grade geography and have found only one pupil and he an unusually brilliant child who showed a real understanding. To give my pupils this tool I have used shadow drawings for June and December with pictures showing activities of the people at different latitudes, 10° apart, and have had the pupils make use of the facts at every possible opportunity in regional units so that they become latitude conscious. A comparison of scores made on a sun behavior test by 8B pupils who had this training for one year and by 8A pupils of the same ability grouping but who had had no special work may be of interest at this point. There are 21 points in the test. The 8B group ranged from 13 to 21 correct with the median number correct 19, and the 8A ranged from 10 to 18 with the median number correct 14. When the 8B group entered as 7B they had taken a similar test in which their scores ranged from 6 to 18 correct with the median score 11.

It is one of our problems then to make sure in all uses of maps that our pupils are properly visualizing the terms and symbols used and when we need to introduce one of

the few new terms or symbols be sure to treat them as we would a foreign language so that our pupils sense the thing symbolized.

Reading Landscape

It is quite common to have pupils read facts from a map concerning the natural features of a region. It is less common and just as essential that the pupils be able to read cultural features, i.e. man-made features, as well, in order that they may actually have a geographic picture of the landscape, i.e. see man's activities in relation to the natural environment. Maps showing distribution of people and other distributions of occupance and use of land may be used as pictures are used to serve as a reconnaissance survey of a region. If a group of maps of India, such as are given in List II, be presented to the pupils, they would be able to list many cultural and natural features that they might see on a trip through a particular part of India, Bengal for instance. In order that each pupil may examine each map I use the plan developed in the geography technique courses at the University of Chicago. The maps are mounted and placed at eye level and spaced at intervals on a wire around the room. Members of the class are numbered to correspond with the number of maps to be used and all the ones, for example, are assigned to map number one, the twos to map number two, etc. The group moves on in one direction around the room to study each of the maps in the set in turn. Some of the items the pupils will read from these maps are:

1. Many people live in this region and their homes are probably close together.
2. There is much land, especially in the eastern half of the area, used for raising rice. This suggests that there are many fields of rice.
3. In the northern part of this region there are many fields of tobacco and tea.
4. In Bengal little land is used for growing cotton, but here and there one might see a few fields of cotton.
5. In many parts of Bengal, especially in traveling to the west, one would see fields of corn, wheat, barley and other grains.
6. Fields of sugar-cane are seen in many places on a journey through Bengal.
7. There are many cattle and goats and a few buffaloes.
8. There is much level land in the valley of the Ganges River.
9. There are three railroad routes which terminate at Calcutta. Railroad tracks, passenger and freight trains, and stations or terminals are suggested.
10. The Ganges River flows through the region and joins the Brah-

maputra to form a broad delta at the eastern end of the plain.

11. Calcutta is a very large city on the Hooghly River, one of the streams through which the Ganges reaches the sea. Buildings in which various kinds of work are being done, many habitations, streets, cars, ships and docks, and railroad terminals are suggested.
12. Many people live in cities along the Ganges. This suggests home in groups, streets, and business centers.
13. Boats might be suggested as means of carrying the passengers and freight upstream and downstream on the Ganges.
14. To the north one may see the high mountains.
15. South of the valley of the Ganges the land is higher than in the plain.
16. The land near the mouth of the Ganges River receives a very heavy summer rainfall.
17. The Ganges valley receives heavy rain in summer and is not entirely without rain in winter as some parts of India are. After reading facts of this sort from the maps the pupils are led to formulate suggested relationships between human activities and natural environmental factors such as:
Dense population of delta lands and intensive use of the land are related to heavy rainfall and broad level delta lands. Such relationships should be considered only as tentative and should be checked carefully in the reading following the reconnaissance survey study to find other factors which help to explain the usage of land by man.

The dot maps of distribution of crops and other uses of land are easily obtained for our use in reading landscape. Isopleth maps showing ratios of various kinds are one of the more recent contributions. This type of map eliminates the difficulty of judging how much of the land is used for a certain crop.

As One Source of Problems

It is often said that it is difficult to get pupils themselves to raise worthwhile geographic problems. This difficulty may be met, in part, by presenting disconcerting data for which pupils may try to find an explanation. Here again maps are an effective "tool." Maps showing distribution and ratios often show such striking human facts for which pupils are prompted to ask the reason and thus have the necessary stimulus for geographic thinking. A number of examples of how maps may be used as one source of problems may be fitting at this point. Maps showing distribution of population and of cities are splendid. We have found our set of wall maps of distribution of population for each continent one of the most valuable tools in our equipment.

In presenting a unit on the Commercial and Industrial Region of Western Europe the pupils were shown the pop-

ulation map of Europe and asked, "What striking facts about population does this map show?" The pupils answered somewhat like this: "There is an area of very dense population and many large cities in almost one stretch." "What kinds of work are suggested to you by these facts?" was next asked. From past experience with the densely populated area of northeastern United States the pupils readily responded, "Commerce and manufacturing." (This area of dense population may be roughly outlined as a large capital A on the map.) "What question do these facts suggest to you?" Pupils will ask, "Why is this the most densely populated part of Europe?" and "Are commerce and manufacturing the most important kinds of work in these lands and why?" The search for facts to satisfactorily solve these problems is a worthwhile geographic activity.

A good physical-political map may be used similarly. Presented a map of Africa the pupils are asked, "How does this map suggest that the Union of South Africa is one of the better developed parts of the continent? "There are more railroads and more cities than in other parts," is the easily read answer. The pupils will ask, "Why is this true?" and be started on gaining an understanding of how people in the Union of South Africa are adjusting themselves to their natural environment in finding a solution to this question.

Given a map of cotton production in the United States the pupils are asked, "What striking fact about cotton production in United States does this map show?" The answers might be, "All the cotton is raised in the southern part of the United States," or "There are three areas in which much more cotton is produced than in the other parts of the south." Pupils will ask, "Why is this true?" and try to find facts to help explain these distributions.

Given a map of corn acreage in the United States the pupils will read the striking fact about where most of the corn is grown and ask the question, "Why do these four states use so much of their land for the raising of corn?"

Some of the various kinds of maps which may be used to select such striking cultural or human items as these are given in List III.

It is important in using maps in geography work to have our pupils read from maps, distributions of cultural features or conditions *first*, of raising questions of the types described concerning the "whys" of human activities evidenced and of reading then those facts concerning distributions and types of natural features or conditions which are helpful in explaining the former. This type of practice will lead to accurate geographic thinking and not develop the sloppy thinking and drawing of conclusions from insufficient data, which have been so common. On the other hand, where maps showing distributions and types of natural features are presented first the pupils are led to raise problems concerning the "whys" of natural phenomena which are *not* the *major* understandings which we are striving to give our pupils. Although it is interesting, for instance, to know why Mediterranean lands have hot, dry summers and mild, rainier winters, in *geography* we are trying to give the pupils the understanding of *how* the people in Mediterranean lands are fitting their ways of living to the kind of land in which they live. Let us, therefore, help our pupils to gain the ability to read from the various kinds of maps the distinctive facts which maps tell and lead them to state clearly geographical problems prompted by these facts.

For the Solution of Problems

Some of the facts concerning the natural environment which may, in part at least, help to explain human items may be read from maps showing distribution and types of natural features, a few of which are given in List IV.

To help to explain the dense population and importance of commerce and manufacturing in Western Europe, maps showing distribution of coal fields and of other minerals, such as iron ore and potash, are useful. Maps of land forms and precipitation are helpful. Of course, there are many factors in the explanation which cannot be read from maps.

In understanding why the Union of South Africa is one of the better developed parts of the African continent a physical-political map will suggest such factors in the explanation as—position outside of the tropics and plateau land which probably is suitable for white settlement.

To help to explain the concentration of the cotton producing area in southern United States, the pupils find from the physical map that there is much level land in this area which may be suitable for agriculture; from the map showing lengths of growing seasons they read that the area has 200 days or more growing season; and from the distribution of precipitation map they find all through the cotton belt the average annual rainfall is not less than 30-40 inches. To be sure there are other factors of much importance to be considered, such as soil conditions, time of rainfall, etc. A timely warning in this connection may be, "Remember not everything can be read from a map."

Position of railroad routes is sometimes partly explainable by passes or breaks in mountain systems which can be read from physical maps showing railroads. In similar ways other natural phenomena maps may be used to read such facts about nature and distribution of these phenomena as function in the explanation of human activities related to them.

Map Expression

Map expression gives pupils themselves the opportunity to show distributions, ratios, routes, regions, and such items that are not merely copied in map language but translated into it. These maps, after they have been drawn, must function in gaining understandings of man's activities in relation to natural environment or they have no place in geographic study and become mere busy work or art exercises. Maps made by the pupils may function (1) in raising problems for study, (2) in helping to answer questions, and (3) in presenting summary type of problems, i. e., showing facts about a region in map language which the pupils should be able to explain from their previous study, i. e., testing exercises.

In making maps pupils need a reasonable amount of help in the manipulation of statistics and, in order to insure accuracy, outline maps should be provided for the work.

Maps made by pupils themselves present disconcerting data which arouse questions in a manner similar to that described previously. Where no map of distribution of population is available, pupils may make a dot distribution map of China for example, to show in a striking way the

concentration of population in China Proper. The pupils will be prompted to ask, "Why don't the crowded Chinese move to other sections of the country?" or "How do so many people make a living in China Proper?" I had an 8A class about to study the geography of their state make isopleth maps showing various ratios, such as, per cent of total land in crops, per cent of crop land in various crops, etc., to raise problems such as "Why is there such a wide variation in the per cent of land in crops in our state?" and the like.

Since most of the natural phenomena maps which the pupils need to use to help answer their questions are available either in their texts or in wall map collections it is not necessary to make this type of map. When physical and climatic maps are made by pupils they are apt to be inaccurate copies of the work of experienced cartographers. However, maps which the pupils make showing distributions of cultural items are helpful in gaining answers to problems. The isopleth maps showing ratios of various crop lands to total crop land in the state not only raised questions, but gave part of the answer to the main question of how are the people of our state using their lands and why. Another example of this use may be given. A map showing by dots the potential and developed water power resources of the countries of Europe may be made to function in understanding the manufacturing conditions of Western Europe.

It has been my experience that the third of the above mentioned uses of pupil-drawn maps has been of greatest value. I shall mention only one or two examples of this type. A map on which the pupils color the countries of Europe according to the value of the manufactures produced in each country provides a good testing question of the summary organization type when the pupils are asked to give all the facts they can to help to explain the rank as shown on their maps. After an eighth grade class has completed a study of the United States and its world relations, they used the data given in the U. S. Commerce Yearbook, a book with which they had become familiar, and made two colored maps, one showing rank of countries in value of our imports from them and the other rank in

value of our exports to them. The pupils were then asked to list all the facts they could to help to explain what their maps showed.

Conclusion

A few words about the place of knowledge concerning location of places in geographic study probably belongs in any discussion on use of maps in teaching geography. Geography is primarily a study of the relationships between human activities and natural environmental factors. A knowledge of the location of places is one of the desirable "by-products" of this study. Our pupils ought to be able to locate accurately those places which are most frequently used in furthering or developing the geographic understandings. It is very essential that pupils know *how* to use reference maps in finding locations and other facts they do not know and can get from maps, and that they have the habit of finding any location they do not know at the time they need it.

Use of Graphs

Graphs are another of the helpful "tools" which the teacher of geography has at her disposal. Graphs may be used in many of the same ways as maps are used in helping pupils to gain understandings of relationships of man to natural environment. They may be used as a source of problems, they may be used to help in answering problems (rainfall graphs), and they may be used to present summary situations. Disconcerting data may be shown on graphs which the teacher has available to present to the class, the teacher may construct the graph before the class, or the pupils themselves may be directed to construct the graph from statistical data. Time does not permit a discussion of various kinds of graphs and of the ways in which they may be used. In all cases, however, the same fundamental principle applies, use graphs only if and when they function in helping one to reach a geographic understanding.

To summarize briefly, a teacher should use maps rather than other sources when ideas of relative position, of size, of shape, and of areal distribution are needed and should choose "tools" in terms of the contribution they can make to the geographic understandings which the pupils are to gain.

A GLASS STOPPER PULLER

By FRANK R. PRATT,

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With the Volstead Act in operation one might doubt the need for a stopper puller these days, but our laboratory needed some sulphuric acid and the glass stopper in our new supply bottle refused to budge by all the known methods of warming and tapping. Previous to this we found two reagent bottles with stoppers cemented in for all time. These contained solutions of potassium and sodium hydroxide. A chemist friend told us we had no chance in the world of removing these stoppers and had better throw the bottles away.

Not wishing to destroy three perfectly good bottles as well as the contents, I set out to construct a glass stopper puller and succeeded so well with this, that it might be of interest to those using reagent bottles to have a description of the puller. This puller removed the above mentioned stoppers. The first one came out when the neck of the bottle was warmed. The other two stoppers came out without warming. Since then several others have been removed without warming.

Figure 1 shows a front view of this puller. The outside shell, A, is a piece of brass tubing four inches long and one and one-fourth inches in diameter. This shell is cut away in both figures to show the internal parts. Figure 1 shows two handles which project out beyond the shell. The upper handle is attached to the screw, B, and the lower is attached to the nut, C, and slides up and down in a slot in case, A, when the screw is turned. This lower handle serves for several purposes. First, it prevents the nut from turning in the shell; Second, it serves as a handle to prevent the entire puller from turning when torque is applied to the upper handle; Third, it serves as an axle for connecting the links, D, to the nut. The lower ends of the links are riveted to the jaws, E, and the jaws are free to turn on the rivets.

Figure 2 is a side view of the puller. The lettering is the same in both figures. This view gives a better idea of the way the jaws grip the handle of the stopper, F. This handle is shown by dash lines inside of the jaws. The jaws project through slots cut in the shell.

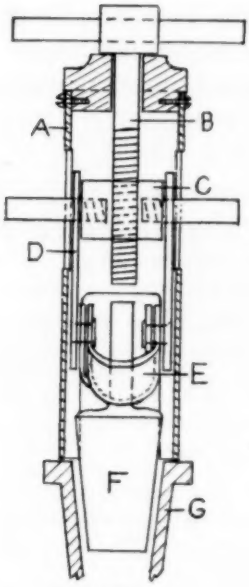


FIG. 1

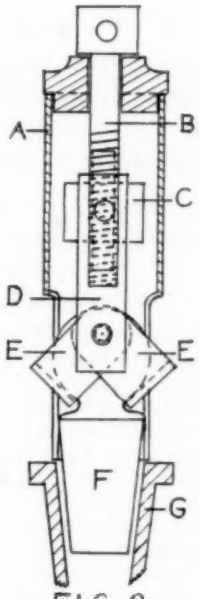


FIG. 2

When the puller is to be attached to the handle of the stopper, the jaws are raised to a horizontal position, and the puller is put down over the handle to the stopper until the case rests on the ring at the top of the neck, G, of the bottle. The handle of the stopper projects up into the jaws. Then the jaws are pushed down until they fit into the neck which connects the handle of the stopper to the stopper as shown in figure 2. If the jaws are too high to fit into the neck they are let down by turning the screw to the left. When the jaws are in place turn the screw to the right until the stopper leaves the neck of the bottle.

If the neck connecting the stopper with its handle is rather small, it is well to put some stress on it and wait a bit and then turn some more. The stopper generally comes on the second turn. If there is danger of breaking the handle off the stopper one can warm the neck of the bottle a little. If the neck of the stopper is rather large the jaws may slip. This is prevented by placing a small clamp across the jaws.

When this puller is used there is no danger of breaking off the neck, G, of the bottle as the stress is on top

of the neck. On account of the links connecting the nut with the jaws there is no possibility of side thrust or torque on the stopper. The pull is entirely in an upward direction, hence the chance of breaking the stopper is very slight. So far this puller has never broken a stopper.

FROM THE SCRAPBOOK OF A TEACHER OF SCIENCE.

By Duane Roller,

The University of Oklahoma, Norman, Okla.

In his capacity of physicist, Kaminsky invented a special apparatus to demonstrate the Boyle-Mariotte law of the resistance of gases. After each demonstration, there were always two or three boys who in a studied whisper would exchange the words "Well done!" Someone would rise and in a doubtful tone inquire: "And who is the inventor of this apparatus?" Kaminsky would answer casually in his frozen falsetto: "I built it." Everybody would exchange glances, and the two-mark boys would admit as loud a sigh of rapture as possible.—*Leon Trotsky, "My Life." Kaminsky was Trotsky's teacher in the realschule.*

A common form of statement of Newton's second law is that the effect of a force is the same whether it acts alone or in conjunction with other forces, meaning, of course, "the accelerating effect." Every schoolboy knows that, in general, the effect of a force is *not* the same whether it acts alone or in conjunction with other forces; two boys on thin ice may produce an effect which is absolutely different from anything which can be produced by the weight of one boy alone.—*William S. Franklin and Barry MacNutt, "Mechanics and Heat."*

Wealth of thought, not wealth of learning, is the thing to be coveted.—*Democritus.*

The only way in which one human being can properly attempt to influence another is by encouraging him to think for himself, instead of endeavoring to instill ready-made opinions into his head.—*Sir Leslie Stephen, English philosopher and man of letters.*

Strange as it may seem, the strength of mathematics lies in the avoidance of all unnecessary thoughts, in the utmost economy of thought-operations.—*Ernst Mach.*

Plato wrote above the door of his school of philosophy, "Let only those with geometry enter here." A similar legend might well appear above the entrance to every laboratory of physics, not primarily because the study of geometry affords a discipline in rigorous thinking, but rather because geometry is itself a branch of physics.—*Prof. Noel Charlton Little, in "College Physics"*

It is an important part of knowledge to know where knowledge ends.—*Prof. A. A. Knowlton, in "Laboratory Manual in Physics."*

Kepler, the legislator of the heavens.—*Prof. Frank Allen, in "The Universe," a popular presentation of relativity*

SCIENCE IN THE TEACHERS COLLEGE

By FRED R. CLARK,

Southeastern Teachers College, Durant, Okla.

For some time our Normal Schools and Teachers Colleges have been in the grip of a tendency to place most of their emphasis on educational method and little if any stress on subject matter. It is not surprising therefore that we are finding that many of our schools are turning out graduates perfectly prepared in method but knowing next to nothing of the subject which they are supposed to teach. It is a sad commentary on our system of education that such a lopsided condition should exist. It is especially notable with the sciences that the training of students in such institutions is sketchy and poorly grounded. Notwithstanding the assertions of certain prominent college professors of Education that it is unnecessary to know much of subject matter if you are well grounded in methods of teaching, the writer believes that you cannot properly teach any subject, be it Chemistry, History, Biology or even Education, without good, solid, basic preparation in that field. Now this does not mean that one must needs be a research chemist in order to teach Chemistry in a high school, but it does mean that frequently a little more solid study of the methods and aims of modern science of this type would not be amiss. A great deal of the current instruction in the sciences as given in the teachers colleges today is definitely of low grade and not up with the times. It is time that this should be improved. We expect that the large Universities and colleges will admit our graduates, but we do not offer work as up to date and thorough as theirs. Some of our schools are turning out students to teach in the sciences who are not as well equipped as similar students were ten years ago. Yet during this period science has made enormous advances in every field of endeavour. We are living in a "scientific" age. Everywhere we are in direct contact with machines, with processes and productions that are of scientific origin and complicated in the extreme. To understand our world of today we must have a good deal of scientific background. The man of the street today knows far more of the nature of science than his ancestors did in the generations before. He needs to know this. Why then are we preparing the

teachers of the next generation in everything but subject matter? Can it be that we are planning to go back to the ages of superstition and academic discussion? Certainly some of the tendencies at present in operation are not at all encouraging. Is it not worthy of note that teachers colleges are contributing little if anything of scholarly or research nature in the field of science, whereas the agricultural colleges, the universities, the arts and science colleges, and various other institutions are turning out great amounts of excellent work?

The writer feels that our schools are not keeping up with the modern thought of the world along scientific lines. Rather we are letting down in adequate preparation and insisting on too many non-essentials. Teachers college graduates may know a great deal of the theories of teaching but if they do not know their subject matter they can teach little of value. It has been said that to be able to teach a dog you must know more than the dog!

At this point the writer wishes to point out that he has no fault to find with Education courses as such. He believes that they need a most prominent place in the teachers college and in fact in all colleges and universities. He believes that especial emphasis should be given to them in the training of prospective teachers. But he does not think that they should supplant the more basic subject matter courses. Rather they should act as an aid to their more ample elucidation.

It is necessary or at least desirable that our science students in the teachers college have not only sound preparation by means of the usual classroom methods but also some experience in the working out of an individual problem or bit of research. It is not necessary that this be a complicated or enduring contribution to thought, but it should be able to give the student an insight to the methods of investigating new fields of knowledge. Book science has a tendency to become platitudinous and pedantic. There is still much in Agassis' assertion that we should "Study Nature, not books." A student teacher who has made some careful studies of a research nature is quite likely to respect and further investigative initiative among his own students when he gets to teaching for himself. There can be little doubt that such

initiative has been one of the outstanding contributions to our progress as a nation among nations. We cannot think of giving up this prized position as one of the foremost countries in scientific lines. Consequently we must work for more emphasis on the sciences not only in the larger colleges but also in the smaller schools and teachers colleges that play so important a part in educating the masses of our people. If we do not train our people to take their proper place in the modern scientific world we cannot expect our modern world to endure. Knowledge is power.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON,

State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor, should have the author's name introducing the problem cr solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

LATE SOLUTIONS

1198. *Sudler Bamberger, Harrisburg, Pa.; Leo Aroian, Fort Collins, Colo.*
1183. *Leo Aroian, Fort Collins, Colo.*
1185-1188, 1192, 1193, 1196, 1198. *Charles W. Trigg, Los Angeles, Calif.*
1185, 1186. *Leo Aroian.*

SOLUTIONS OF PROBLEMS

Editor.—Persons sending in solutions should read carefully the instructions about the form of the solutions and the ink-drawn figures. Many times, a good solution is received, but poorly arranged and no India-ink figure given.

1199. *Proposed by O. T. Snodgrass, Columbia, Mo.*

For the equation $x^y = y^x$

Find $\frac{dy}{dx}$ to be $\frac{y^2(\log x - 1)}{x^2(\log y - 1)}$

Solution by Arthur J. Kavanagh, Washington, Conn.

Evidently $y \log x = x \log y$

or $\frac{\log y}{y} = \frac{\log x}{x}$

whence $\frac{1 - \log y}{y^2} \cdot \frac{dy}{dx} = \frac{1 - \log x}{x^2}$

or $\frac{dy}{dx} = \frac{y^2(\log x - 1)}{x^2(\log y - 1)}$

This derivation assumes $x \neq 0$, $y \neq 0$.

Also solved by Charles W. Trigg, Los Angeles, Calif.; M. C. Erwin, Reynolds, Ind.; H. R. Harold, Tonkawa, Okla.; W. E. Buker, Leetsdale, Pa.; E. C. Kennedy, El Paso, Texas; John E. Bellanrds, St. Nazianz, Wis.; Daymond J. Akin, Shawneetown, Ill.; Ralph Newman, Clarksville, Ark.; M. Irving Chriswell, Buffalo, N. Y.; R. T. McGregor, Elk Grove,

Calif.; Sudler Bamberger, Harrisburg, Pa.; Leo A. Aroian, Fort Collins, Colo.; Harold Venske, Solvang, Calif., and Günther Wunsche, Dresden, Germany.

1200. *Proposed by R. T. McGregor, Elk Grove, Cal.*

Evaluate $(\frac{\tan x}{x})^{\frac{1}{x^2}}$ when $x=0$.

Solved by M. C. Erwin, Reynolds, Ind.

Since $\underset{x \to 0}{\text{Lim}} (\frac{\tan x}{x}) = 1$, there results the indeterminate form 1^{∞} when $x=0$ is placed directly for x.

(1) Write $(\frac{\tan x}{x})^{\frac{1}{x^2}} = (\frac{\sin x}{x \cos x})^{\frac{1}{x^2}}$ and expand $\sin x$ and $\cos x$ in powers of x.

(2) Then $\underset{x \to 0}{\text{Lim}} (\frac{\tan x}{x})^{\frac{1}{x^2}} = \underset{x \to 0}{\text{Lim}} (\frac{\sin x}{x \cos x})^{\frac{1}{x^2}}$

(3) $= \underset{x \to 0}{\text{Lim}} \left(\frac{x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots}{x - \frac{x^3}{2} + \frac{x^5}{4} - \cdots} \right)^{\frac{1}{x^2}}$

(4) $= \underset{x \to 0}{\text{Lim}} \left(1 + \frac{x^2}{3} + \frac{2x^4}{15} + \text{higher powers of } x\right)^{\frac{1}{x^2}}$

(5) $= \underset{x \to 0}{\text{Lim}} \left[1 + \frac{x^2}{3}(1 + \frac{2x^2}{5} + \cdots)\right]^{\frac{1}{x^2}}$

In (5) the series in parentheses is in ascending powers of x. Since its first term is unity its limit is 1 when $x \to 0$. Represent this series by C. The (5) may be written

(6) $= \underset{x \to 0}{\text{Lim}} (1 + \frac{cx^2}{3})^{\frac{1}{x^2}}$, which may be written in the form:

(7) $\underset{x \to 0}{\text{Lim}} (\frac{\tan x}{x})^{\frac{1}{x^2}} = \underset{x \to 0}{\text{Lim}} \left[(1 + \frac{cx^2}{3})^{\frac{3}{cx^2}} \right]^{\frac{c}{3}}$

But the $\underset{x \to 0}{\text{Lim}} (1 + \frac{cx^2}{3})^{\frac{3}{cx^2}} = e$, and since limit of $c=1$ we have

(8) $\underset{x \to 0}{\text{Lim}} (\frac{\tan x}{x})^{\frac{1}{x^2}} = e^{\frac{1}{3}}$

Also solved by F. A. Batter, Jr., San Jose, Calif., H. R. Harold, R. T. McGregor, Elk Grove, Calif., Arthur J. Kavanaugh, Washington, Conn., E. C. Kennedy, El Paso, Texas, and Leo H. Aroian, Fort Collins, Colo., Günther Wunsche, Dresden, Germany.

Three incorrect solutions, each assuming that $1^{\infty} = 1$, were received.

1201. *Proposed by Lester Dawson, Wichita, Kans.*

Factor $(x+y)^5 + (x+z)^5 + (y+z)^5$

No satisfactory solution received.

1202. *Proposed by Clyde Bridges, Walla Walla, Washington.*

Find the magnitude of a trihedral angle of a regular tetrahedron. See problem 1133.

No satisfactory solution received.

1203. *Proposed by Nathan Altshiller-Court.*

The common perpendicular of two opposite edges of a tetrahedron is perpendicular to the lines joining the mid-points of the other two pairs of opposite edges of the tetrahedron.

Note: A solution by use of vector methods has been offered, but the Editor prefers to offer a solution by the ordinary methods of geometry.

1204. *Proposed by a Student, Eldon, Mo.*

Solve for x and y in the pair of equations:

$$\left.\begin{array}{l} \dfrac{x^2+y^2}{2x}=a \\ \dfrac{x^2+y^2}{2y}=b \end{array}\right\} (1)$$

Solved by F. A. Butter, Jr., San Jose, Calif.

$$\left.\begin{array}{l} x^2+y^2=2ax \\ x^2+y^2=2by \end{array}\right\} (2)$$

$$\therefore ax=by \quad (3)$$

$$\therefore y=\frac{ax}{b}$$

Substitute this in the first equation of (2): $x^2+\dfrac{a^2x^2}{b^2}=2ax$

$$x\,(a^2x+b^2x-2ab^2)=0.$$

Hence we have 2 roots:

1) $x=0$, which satisfies (2) but not the original form (1) and hence must be rejected. If $x=0$, then $y=0$, whence a and b in (1) are indeterminate $=\dfrac{0}{0}$. Further, we see from (3) that if either $a=0$ or $b=0$, it follows that either $x=0$ or $y=0$, either of which necessitates the other.

Hence: $a\neq 0$, $b\neq 0$, $x\neq 0$, $y\neq 0$ in (1).

(2) Solution: $x=\dfrac{2ab^2}{a^3+b^3}$; $y=\dfrac{2a^2b}{a^3+b^3}$

Note: Form (2) represents 2 circles intersecting at the 2 points $\left(\dfrac{2ab^2}{a^3+b^3}, \dfrac{2a^2b}{a^3+b^3}\right)$ and (0,0).

Also solved by Samuel M. Cohn, Philadelphia, Pa.; Victor Cox, Wichita, Kans.; Harold R. Howard, Tonkawa, Okla.; W. E. Buker, Leetsdale, Pa.; Sister M. Cherlotte, Boise, Idaho; Daymond J. Akin, Shawneetown, Ill.; W. E. Batzlea, Battle Creek, Mich.; Leo Aroian, Fort Collins, Colo.; Albert Whiteman, Philadelphia, Pa.; R. T. McGregor, Elk Grove, Calif.; and Charles W. Trigg, Los Angeles, Calif.; Gunther Wunsche, Dresden, Germany, and Andrew Sobczyk, Duluth, Minn.

Three incorrect solutions were received.

PROBLEMS FOR SOLUTION

1217. *Proposed by P. H. Nygaard, Spokane, Wash.*

A rabbit is running at a constant speed of 30 ft. per sec. along the circumference of a circle whose radius is 50 ft. A dog starts at the center of the circle and runs directly toward the rabbit at a constant speed of 60 ft. per sec. Find the time required for the dog to catch the rabbit. A solution using differential equations is desired.

1218. *Proposed by Albert Schwartz, Brooklyn, N. Y.*

How many sides have the polygons the number of whose diagonals is a square?

1219. *Proposed by Guy C. Lentini, Boston, Mass.*

A sphere of radius a is placed in a cube of edge $2a$ with 8 other spheres. If each of the eight small spheres is tangent to 3 faces of the cube and

externally tangent to the sphere of radius a, find the radius of a small sphere.

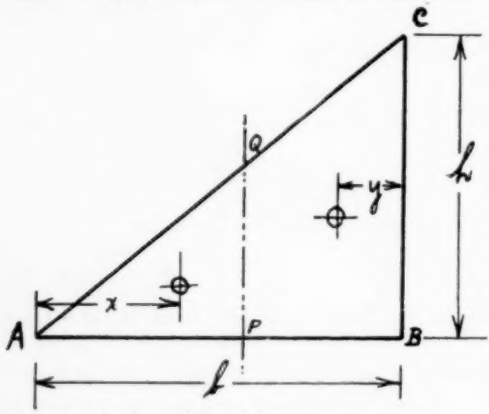
1220. *Proposed by J. H. Dauby.*

If a horse is tied with a rope 100 feet long to a corner of a barn 40 feet square, over how much ground can he graze?

1221. *Proposed by Günther Wunsche, Dresden, Germany.*

Find the condition, which must exist between two numbers a and b, if the arithmetical, the geometrical, and the harmonical mean of them are pythagorean numbers.

1222. *Proposed by F. H. Wade, Chicago, Ill.*



Problem: Given a triangle with horizontal base b and vertical altitude h, divided into two parts by the vertical line PQ. Required the position of PQ so that the moment of area of left-hand part with respect to A be equal to the moment of area of right-hand part with respect to B. That is, if x and y are the distances to the respective centroids, required to find PQ so that area APQ:x = area BPQC:y.

This result is of importance, in bending of beams.

The Editor is pleased to make special mention as suggested in the December issue for high schools whose students as individuals or classes or clubs make noteworthy contribution to this Department. Teachers are urged to make report to the Editor when some extensive study has been made in their high school problems submitted for solution.

In this issue we are glad to make honorable mention as follows:

1204. *Donald Reich, James MacLean, and Russel Pottinger from the LaSalle, Illinois, High School; Walter Nebel, Stanford, Montana; Vernon Godsey, Robert Williams, Harold Riles, and Carl Morig, Spokane, Wash.*

THANKS FOR THE COMPLIMENTS

From Dresden, Germany, comes an appreciative note from Günther Wunsche, a Professor of Mathematics in the Technical University: "For a long time I have been a much interested reader of the periodical, SCHOOL SCIENCE AND MATHEMATICS. With greatest interest I follow up your problem department.

From the Standford High School, Standford, Montana:

"My high school mathematics classes are becoming quite interested in the problem department and we look forward for each issue of SCHOOL SCIENCE AND MATHEMATICS.

From the Reynolds High School, Reynolds, Ind.:

"SCHOOL SCIENCE AND MATHEMATICS is a very valuable journal, both for its direct help and its inspiration for one engaged in active teaching."

From a student at the University of Illinois:

"SCHOOL SCIENCE AND MATHEMATICS is one of the best magazines I have seen in regard to science and problematical articles."

NEW MAPS OF CHICAGO REGION

By D. L. Carroll

The most popular set of topographic maps ever issued in this country, covering more than a thousand square miles of the Chicago area, is now available as a result of a resurvey of the district recently completed by the United States Geological Survey, in cooperation with the Illinois Geological Survey. There are twenty maps in the set, each of which covers more than fifty square miles of territory on the scale of about 2½ inches to the mile, representing the highest standard of accuracy ever attained in this type of mapping work.

The degree of detail on these maps is astonishing. Every building in the area is shown, for instance, and in the case of the larger buildings the true ground plan shape is represented. Churches, schools, lighthouses and other special buildings can be easily identified on the maps. All streets and boulevards are shown, as are surface and "L" lines, railroads, railroad yards, spur tracks and switches. All occurrences of water are shown in blue, so that the shore of Lake Michigan can be easily traced, and all harbors, lagoons, small lakes and ponds, flooded quarries, rivers, creeks, canals, and even marshes, can be spotted at once. Parks, cemeteries, golf courses, race tracks and other areas of open ground are prominent features of the maps. The contour interval used in making this set of maps (5 feet) is so small that almost every noticeable irregularity of the land surface shows up quite plainly and in minute detail, even the bunkers on golf courses throughout the area are faithfully plotted.

By studying the contour lines on these maps it is comparatively easy to pick out the hills, hummocks and "kettle holes" of the glacial moraines that remain in the vicinity of Chicago, and which mark the stages of retreat of the ice sheet which occupied the Lake Michigan basin thousands of years ago. The outlets, beach lines, spits and sand bars of the ancient glacial Lake Chicago, which covered a much greater area than the present Lake Michigan, are very evident. So are the lake bluffs of today and the excarpment that marks a bluff cut by the waves when the lake stood 40 to 50 feet higher than it does now. The highest of the ancient beaches is about 50 feet above the present surface of Lake Michigan, and there are several lower beach systems . . . all easily traceable on the maps.

Many of the events in the natural history of the area can be read by a study of the contour lines and other symbols. For example, the Highland Park map shows a lake cliff about 100 feet high, which was formerly being continually undermined by the waves and southward-flowing shore currents. Short breakwaters built out from the shore, as shown on the map, now arrest these activities and are actually causing these same currents and waves to deposit material on the north sides of the breakwaters, thus turning the forces of nature from destructive work to constructive work at that point. These maps are then extremely useful to students of the natural history of this area, as well as to everyone interested in the multifarious activities of the "Windy City" and its neighboring towns and villages.

The maps may be secured in any quantity at the standard prices of state and government maps, by addressing the Chief, State Geological Survey, Urbana, Illinois, or the Director, U. S. Geological Survey, Washington, D. C.

PHYSICS BY AN INDIVIDUALIZED METHOD

By RALPH C. HOLLIS,
Senior High School, Waltham, Mass.

After three years of trial, we feel that we can state very emphatically that *individualized* instruction *is* possible in teaching physics. It is possible, whether classes contain as many as thirty, or as few as eight pupils. It works well with rapid or slow pupils. It produces results as good as, or better than, other methods.

Our method of individualizing the work is not a complicated one, and has proved easy to use. The work of the year is divided into units, each unit being so chosen that it may be understood as a whole. The units we have used successfully are: I. Mechanics of Liquids, II. Mechanics of Gases, III. Molecular Physics, IV. Force, V. Motion, VI. Work, Power, and Energy, VII. Machines, Friction, and Efficiency, VIII. Heat—Thermometry and Expansion, IX. Heat Capacity and Change of State, X. Heat—Transmission and Distribution, and relation to Work, XI. Sound, XII. Light—Illumination and Reflection, XIII. Light—Refraction and Dispersion, XIV. Electricity—Production and Transmission, XV. Electricity—the Magnetic, Chemical, and Heating Effects, XVI. Electricity—Induced Currents, Generators and Motors.

Unit assignments are made by the now familiar method of assignment sheets. Each pupil is provided with an assignment sheet covering the work of a unit, which usually requires about two weeks' study. This sheet states definitely: (1) The time allowed for the work, with the date of the unit test, (2) The objectives of the assignment, carefully stated, (3) The learning exercises to be done in order to obtain the abilities desired, (4) A division of the work in such a way that the more rapid pupils may do more, and more difficult, work than the slower pupils (see specimen assignment sheet).

SPECIMEN ASSIGNMENT SHEET

Assignment for January 19 to January 30, 1931. (Test Jan. 30.)
Unit IX. Heat—Heat Capacity and Change of State.
Objectives: This unit is intended to enable you to understand, to explain, and to apply:

1. Heat Measurement, using calorie and British Thermal Unit.
2. Specific Heat.

3. Heat of Fusian (melting), and Heat of Vaporization.
4. Laws of evaporation and boiling.
5. Principles of refrigeration.
6. Humidity, dew point, relative humidity, and the relation of heat to weather.

Learning Exercises: To gain the above abilities, you will do the following:

1. Study: Dull, pages 280-313.
   Black and Davis, pages 234-262.
2. Memorize:
   Definition of Calorie.
   Definition of British Thermal Unit.
   Definition of Specific Heat.
   Definition of Heat of Fusion.
   Definition of Heat of Vaporization.
   Definition of Relative Humidity.
   Definition of Dew Point.
   Laws of Evaporation.
   Laws of Boiling.
3. Solve Problems:
   Credit toward a grade of C:
   Dull, page 291—1, 2, 3, 4, 5, 9, 11.
   Dull, page 312—1, 2, 4, 5, 10.
   Credit toward a grade of B: those above, and
   Dull, page 291—6, 7, 8, 10, 12.
   Dull, page 312—3, 6, 7, 8, 9, 11.

NOTE: Remember that a grade of A is given only for the highest quality of work.

The work is further individualized by having no recitation periods as ordinarily used. Instead, we carry on the daily classroom work with individuals at the teacher's desk. A pupil is called to the desk if the teacher knows he has a definite need, or comes of his own accord when he encounters difficulties. The teacher, at this time, discusses all difficulties, whether concerned with problems, theory, or practical applications. This method ensures that the pupil so instructed really understands, and really overcomes his difficulty. The teacher points out the pupil's mistakes in problems, including mathematical errors. The discussed papers are placed in a file (inaccessible to pupils), and the pupil repeats the problems he did incorrectly, until he has them right. The papers are returned to the pupils the day before the test, in order that they may review their corrected work.

No grade is given for problems, and the only limit of time for passing them in is the date of the unit test. We make the pupils understand that they will be unable to do well (that is, to receive more than a C) in the test, unless the practice problems have been faithfully worked. Tests are so designed that the pupils not preparing for college may receive a passing grade without working the more difficult problems. Thus the problems are practice work only,

and are largely optional. Because the pupils do not feel compelled to do a certain amount of problem work, or to do it in a set time, most of them do not mind solving a considerable number of carefully selected problems. The ambitious student usually asks for more! This plan also seems to reduce mere copying to a minimum. Pupils *do* work together, but seem, in most cases, to benefit from the mutual help. Many have been known to disagree, and to pass in different solutions of a problem they have discussed together, waiting eagerly for the teacher's decision as to who was right.

We allow pupils who do steady work, of good quality, to select their own extra study material, either in problem solution, in extra experiments, or in advanced work. At the beginning of the school year, statements are taken from the pupils regarding their intentions for the future, as to college, normal school, or work. These statements are checked quarterly, in case of change of mind on the part of pupil or parent. Those definitely not preparing for college may choose work more on a practical trend, may omit some of the problems, and are in general more free of choice than those who are preparing to go to college. The latter, as we know only too well, must keep up their work at a truly fearful pace, in order to complete their requirements.

Because of present college entrance requirements, the class is kept at approximate "even front," and College Entrance Board "ground to be covered" is cared for, by the minimum time limit on each unit, and by testing at the end of each unit in such a way that the teacher knows whether each pupil is, or is not, fitted for certificate to college, or for taking the College Entrance Board Examination.

The laboratory work, for which our school is exceptionally well fitted, is made to coincide closely with the unit being studied. A list of so-called "regular" experiments (about thirty-five in number) is provided, all of which are usually performed individually, or in small groups, by the majority of the pupils. In addition to these, we have an equally long list of possible "supplementary" experiments, from which extra work may be chosen. Pupils who do exceptionally good and pains-taking work in the laboratory are allowed to do experiments involving more difficult manipulation and more delicate apparatus.

Present marking requirements are met by grading the unit tests, and the laboratory experiments, as well as by giving a test, of College Entrance Examination Board type, at midyear, and at the end of the year. We wish that we could do away with this grading, but it seems to be impossible to do so at present. Grading does not seem to be necessary in order to make the pupils work.

Although the physics course seems to be largely self-motivated, because it is entirely elective, we provide some additional "driving force" by introducing each unit with a demonstration lecture. This lecture-demonstration is a short summary of the work of the unit. It serves to arouse curiosity and interest, and to start thought and questions. The interest is kept up by leaving the demonstration apparatus where the pupils may study it themselves, the teacher answering and asking questions whenever this is helpful. We also sustain the interest by occasional short general discussions of things of special interest, or of general difficulty. We find that the pupils like very much our bulletin boards, on which clippings (advertisements, pictures, articles) concerned with everyday practical applications, or with modern theory and discoveries, are displayed.

The question, "What are the results of this method?" may now be answered. We find the results good, as shown in several ways. Few pupils fail; the only causes of failure being protracted absence (although this method reduces the absence difficulty greatly), or absolute refusal to do any work. Many pupils (a rather extraordinarily large proportion) attain certificate grades of B or A, these being given only on proven ability with trial College Entrance Board Examinations. Many pupils, in the three years that we have tried this method, have had fine success with their actual entrance examinations, several receiving highest honors. Objective tests at the end of the year show better understanding than that shown by similar classes, taught by the daily recitation, "whole class" method, when given the same test. We have, in short, found the method so efficient, that we plan to continue its use. No doubt, improvements may be made, and we should very much enjoy and appreciate comments and criticisms from our fellow teachers.

SHALL MODERN PHYSICS BE INCLUDED IN THE HIGH SCHOOL COURSE*

ABSTRACT OF THE REPORT OF AN INVESTIGATION BY G. W. DURFLINGER

The purpose of this investigation is to furnish scientific evidence which will aid in the determination of what topics from the field of modern physics should be included in a high school course in physics. Questionnaires were sent to 503 high school physics teachers in every state of the United States and Washington, D. C., and 78 college and university physics teachers selected at random throughout the United States. Returns were received from 211 high school teachers and 33 college and university teachers.

This investigation assumes that the school teachers are one group which should be consulted in the reorganization of any curriculum. The high school teachers of physics teach the students the physics material directly and the college and university instructors are teaching present and future teachers of the high school students. So the opinions of these two groups are valuable.

The investigation is divided into two parts: first, that of obtaining the subjects of modern physics which might be taught and second, that of ascertaining the opinions of the aforementioned two groups of teachers on what should be taught the high school pupils.

The whole investigation was carried on by the writer at Colorado State Teachers College. The first part consisted of analyzing 27 books including 6 college physics textbooks. This analysis revealed 88 topics of modern physics to which these books gave space. These topics were then arranged under eight different headings and placed into a questionnaire. These headings are ionization, radioactivity, atomic structure, X-rays, photoelectric effect, recent additions to the electromagnetic wave theory, quantum theory, and theory of relativity.

The study sought to find which of these 88 topics the teachers thought should be taught thoroughly, just explained, or not studied at all. Also the study sought to find if these subjects should be in the textbook and laboratory experiments. The study also sought information regarding the subject matter taught at present and the textbooks used.

The two groups of teachers interrogated have indicated that in general the topics found under the headings ionization, radioactivity and the electromagnetic wave theory are the ones they most desire should be taught. The topics under the headings X-rays, photoelectric effect, quantum theory, and the theory of relativity are in general the ones these teachers think ought not to be taught. The atomic structure has about half of its items in the list that should be taught and half in the list that should not be taught.

None of the 88 topics listed did the teachers want in the laboratory experiments.

Over 91 per cent of the high school teachers said they used supplementary physics material in addition to the textbook.

Only 43.33 per cent of these teachers said they use all the material of the textbook.

The three reasons ranking highest among those for not completing the textbook are, first, lack of time, second, material too difficult, and third, not in state syllabus.

*Durflinger, G. W. *A Determination of the Material From the Field of Modern Physics Which Should be Included in the High School Physics Course.* Unpublished Master of Arts Thesis, Colorado State Teachers College, Greeley, 1931.

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BOOKS RECEIVED

Qualitative Chemical Analysis of Inorganic Substances with Explanatory notes by Olin Freeman Tower, Hurlbut Professor of Chemistry in Adelbert College of Western Reserve University. Sixth Edition, Revised. Cloth. Pages xv+92. 15x23 cm. 1932. P. Blakiston's Son and Company, Inc., 1012 Walnut Street, Philadelphia, Pa. Price $1.50.

Lives in the Making by Henry Neumann, Leader of the Brooklyn Society for Ethical Culture. Cloth. Pages xiv+369. 12.5x19 cm. 1932. D. Appleton and Company, 32 West 32nd Street, New York, N. Y. Price $2.25.

Brief College Algebra by William L. Hart, Professor of Mathematics in the University of Minnesota. Cloth. Pages viii+334+30. 13x19.5 cm. 1932. D. C. Heath and Company, 285 Columbus Avenue, Boston, Mass. Price $1.96.

The Introductory Biological Sciences in the Traditional Liberal Arts College by George E. Nelson, Instructor in Biology and Assistant Librarian, College of the City of New York. Contributions to Education, No. 501. Cloth. Pages viii+135. 15x23 cm. 1931. Bureau of Publications, Teachers College, Columbia University, New York City. Price $1.50.

General Mechanical Drawing for Beginners by Robert C. Woellner, Assistant Professor, School of Education, University of Chicago, formerly Principal, University of Chicago High School, and Eugene C. Wittick, Instructor of Mechanical Drawing, University of Chicago High School. Cloth. Pages ix+116. 14.5x23 cm. 1932. Ginn and Company, Number 15 Ashburton Place, Boston, Mass. Price $1.12.

The Unit Mastery Arithmetics by John C. Stone, Professor of Mathematics, State Teachers College, Montclair, New Jersey and Clifford N. Mills, Professor of Mathematics, State Normal University, Normal, Illinois. Illustrated by Marion C. Allen. Cloth. 13x19 cm. Primary, ix+493 pages, price 50 cents; Intermediate, x+486 pages, price 84 cents; Advanced, viii+520 pages, price 88 cents. Benj. H. Sanborn and Company, 221 East 20th Street, Chicago, Ill.

Modern Junior Mathematics by Walter W. Hart, Associate Professor of Mathematics, School of Education and Teacher of Mathematics, University of Wisconson, Madison, Wisconsin. Cloth. 12.5x 18.5 cm. 1932. Book One, Grade Seven, 255 pages, list price 84 cents. Book Two, Grade Eight, 271 pages, list price 88 cents. Book Three, Grade Nine, 351 pages, list price $1 28. Teachers Manual for Book One, 38 pages. 13x18.5 cm. List price 20 cents. Teachers Manual for Book Two, 32 pages. 13x18.5 cm. List price 20 cents. D. C Heath and Company, 285 Columbus Avenue, Boston, Massachusetts.

Recent Advances in Physics by F. H. Newman, Professor of Physics, University College of the South-West of England, Exeter. 51 Illustrations. Cloth. Pages ix+378. 13x20 cm. 1932. P. Blakiston's Son and Co., Inc., 1012 Walnut Street, Philadelphia, Pa. Price $4.00.

Eleventh Year Mathematics, Intermediate Algebra and Trigonometry by Joseph B. Orleans, Head of Mathematics Department, George Washington High School, New York City and Hallie S. Poole, Head of Mathematics Department, Lafayette High School, Buffalo, New York. Cloth. Pages x+261. 13.5x19.5 cm. 1932. D. C. Heath and Company, 285 Columbus Avenue, Boston, Massachusetts. Price $1 32.

Fundamentals of Insect Life by C. L. Metcalf, Professor of Entomology in the University of Illinois and W. P. Flint, Chief Entomologist, Illinois State Natural History Survey, and Entomologist of the Illinois Agricultural Experiment Station. First Edition.

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Cloth. Pages xi+581. 14.5x23 cm. 1932. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, N. Y. Price $4.00.

PAMPHLETS RECEIVED

The Work of the Institute of School Experimentation by Otis W. Caldwell, Director and the Staff of the Institute of School Experimentation. A reprint from Teachers College Record, Vol. xxxiii, No. 2, November, 1931. 35 pages. 17x25 cm. Bureau of Publications, Teachers College, Columbia University, New York City.

The Junior High Schools, Report of the Superintendent of Schools to the Board of Education of the City School District of the City of Cleveland for the School Year 1930-31. 230 pages. 15x23 cm. Published by the Board of Education, Divisions of Publications, Cleveland, Ohio.

The Engineering Field, Bulletin of Vocational Guidance, General Series, Vol. 23, No. 2A, March, 1931. 24 pages. Illustrated. School of Mines and Metallurgy, University of Missouri, Rolla, Missouri. Copies may be secured without cost.

Women in Industry, A Series of Papers to Aid Study Groups by Mary Elizabeth Pidgeon. Bulletin of the Women's Bureau, No. 91. xiii+79 pages. 14x23 cm. Women's Bureau, U. S. Department of Labor, Washington, D. C.

Helps for Schools in Celebrating the George Washington Bicentennial prepared by Florence C. Fox, Associate Specialist in Elementary Education, U. S. Office of Education. iii+23 pages. 15x23 cm. 1931. Superintendent of Documents, Washington, D. C. Price five cents.

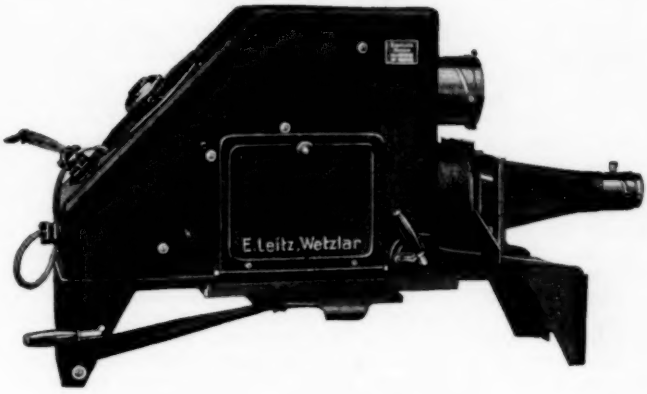
Report of a Study of the Secondary Curriculum by the Secondary Education Board Committees on the Curriculum Study. Howard T. Smith, Chairman, and Homer W. LeSourd, Secretary. 237 pages. 15 5x23.5 cm. 1932. Office of the Board, Milton, Massachusetts.

BOOK REVIEWS

Instructional Tests in Plane Geometry by Florence Bishop, Chairman of Mathematics Department Central High School, Flint, Michigan, and Manley E. Irwin, Assistant Director, Curriculum Research, Public Schools, Detroit, Michigan. Paper. Pages x+62. 1932. World Book Co., Yonkers on Hudson, N. Y.

This booklet contains a series of thirty-six tests covering the various phases of the subject of plane geometry. These tests are the result of a testing experiment covering several years in the schools in Flint, Michigan. Norms have been established for them and are contained in the book. Each test covers a definite topic, such as, Congruent Triangles; Parallel Lines and Converses; Angle Measurement; Proportional Lines, etc. In choosing the subject matter to use in the tests the authors have been guided by the various state and city courses of study, the recommendations of the Report of the National Committee on Mathematics Requirements, and the College Entrance Examination Board.

The types of questions used are completion, matching, true false, short answer, completing proofs, construction, and miscellaneous. They require little writing on the part of the pupil and are easily scored. The tests are to be given under proper time limits and printed directions and checked with the use of a key, if the norms are to be used. The authors have attempted to arrange the subject matter in topics to conform most nearly to the order which is followed by the majority of textbooks. Thus the organization of the material in the textbook a teacher is using will in a measure determine the usefulness of the test to him. The tests are quite complete and carefully made and should prove useful both as a measure of the efficiency of instruction and for a diagnosis of pupil difficulty. G. E. Hawkins.



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Arithmetic for Today by Robert F. Anderson, Professor and Head of the Department of Mathematics, State Teachers College, West Chester, Pennsylvania, and George N. Cade, Professor of Education and Director of the Training School, College of Education, University of Arkansas. Book One, pp. vi+313; Book Two, pp. vi+314; Book Three, pp. vi+346. Silver, Burdett and Company, 1931.

This three-book series is designed for use in Grades III to VIII, each book containing the work for two grades. The books are organized on a spiral plan. The first book treats the four operations with integers and fractions and contains a considerable amount of work with measures. The second book gives a more extended treatment of fractions and presents decimals and percentage. It also introduces areas of plane figures. About one-third of Book Three is devoted to business applications, including such topics as banking and insurance. There is a brief chapter on literal number and the equation, and two chapters are devoted to measurement of plane and solid figures and a further treatment of dry and liquid measure. The remainder of the book is given to ratio and proportion and a review of the fundamental processes. Occasional exercises in reading graphs are included. Some review and test material is provided in each book.

In many instances the explanations are clearly presented and the exercises are arranged with definite reference to the steps of difficulty. In other instances this is not the case and one is impressed by the meagerness of the concrete illustrative material used in introducing new concepts. It would seem that the effectiveness of the texts might have been increased by more consistent use of the inductive approach and by devoting more attention to the motivation of the learning of new processes through the presentation of situations showing the need for those processes. Lenore John.

Craftsmanship in the Teaching of Elementary Mathematics, by F. W. Westaway. Pages xvi+665. 15x20 cm. 1931. Blackie and Son, Ltd. Price 15 shillings.

This book written by a former English Secondary School inspector and designed especially for the guidance of the novice in the teaching of mathematics contains much of interest and value to the experienced teacher. It deals with a large variety of topics of a mathematical character. In the fifty chapters the author presents much interesting material and methods of teaching it.

We mention a few of the topics discussed in order to have the reader get some realization of the variety of the material treated. They are as follows: Arithmetic, which includes exponents and logarithms; Algebra; Geometry, Plane and Solid; Projection, Orthographic and Radial; Trigonometry, Plane and Spherical; Calculus; Mechanics; Astronomy; Mathematics in Biology; Non-Euclidean Geometry; Philosophy of Mathematics; Mathematics for Girls; School Mathematical Library. J. M. Kinney.

The Ingenious Dr. Franklin, Selected Scientific Letters of Benjamin Franklin. Edited by Nathan G. Goodman of Germantown, Pa Board. Pages xi+244. 15x20.5 cm. 1931. University of Pennsylvania Press, 3438 Walnut Street, Philadelphia, Pa. Price $3.00.

Few Americans have been great in so many fields as Benjamin Franklin and all school boys and girls should know more about him. This book of Franklin's letters is suitable for supplementary reading in literature courses and in science classes. The rich humor of his letter to *The Journal of Paris* on "Daylight Saving" will be appreciated by city youngsters. Everyone will want to read the kite story as he told it to Peter Collinson, October 19, 1752. His letters on cold air baths, magic squares, the Gulf Stream, bifocal glasses, restoring life by sun rays, and some fifty others filled with interesting observations and speculations will give instruction and entertainment to every member of the family. G. W. W.

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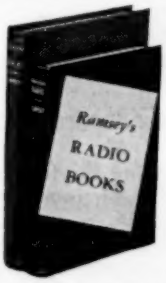

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Mammalian Anatomy, with Special Reference to the Cat by Alvin Davison, Ex-Fellow of Princeton University; Professor of Biology in Lafayette College. Fifth Edition, revised by Frank A. Stromsten, Associate Professor of Animal Biology, State University of Iowa. Cloth. Pages xiv+311. 141 Illustrations. 14x21 cm. 1931. P. Blakiston's Son and Company, Inc., 1012 Walnut Street, Philadelphia, Pennsylvania. Price $2.50.

There are several books on the market that cover the subject of mammalian anatomy, but this is, perhaps, one of the most useful from the point of view of both student and instructor. The style of the book, as with the preceding editions, remains simple and free from confusing technicalities that often tend to obscure the real aims of comparative anatomy. Accuracy has not been sacrificed for the sake of simplicity. Dr. Stromsten has enlarged several sections to advantage. The ductless glands have received additional attention, as has the nervous system. Especially valuable to elementary students are the laboratory studies and suggestions at the end of each chapter. Adequate information as to the preservation of material is found in the introduction. The book contains 273 pages of text, very well illustrated, and a glossary of ten pages. For a general treatment of the subject this book should be found useful in the great majority of courses in comparative mammalian anatomy. Jesse F. Schuett.

Examples in Plane Trigonometry by Winfield M. Sides, Department of Mathematics, Phillips Academy, Andover, Mass. Pages vii+79. McGraw Hill Book Co., New York. 1931.

This book is designed to meet the requirements of those teachers who are in need of supplementary problems in trigonometry. There are 680 carefully graded and selected exercises which appear under their proper heading. The teacher will also find many excellent problems which can be used to stimulate the superior pupil. The book should find a place in the library of every teacher of trigonometry.

C. A. Stone.

THE CENTER OF POPULATION OF THE UNITED STATES

By L. S. Shively,

Associate Professor of Mathematics, Ball State Teachers College, Muncie, Ind.

Preceding the taking of the 1930 census of the United States, there was made an investigation which resulted in a predicted location of the nation's new center of population. This prediction was based upon estimated populations of the various states, these in turn being dependent upon yearly estimates of such populations for each year following 1930, furnished by the department of Commerce. The results, published in this journal*, showed a predicted movement of about 13.5 miles, almost wholly westward, of the center of population during the decade 1920-1930.

The announced location of the 1930 center of population by the Bureau of the Census places it several miles northwest of Linton, Indiana, at a point approximately 15 miles in a southwesterly direction from the predicted location. The explanation of this wide difference can readily be seen to lie in the deviations of the estimates of the Department of Commerce from the actual populations of the states.

A complete discussion in which these deviations are exhibited for all the states is hardly necessary or desirable. Suffice it to say that for the six most populous states of the union, the variations of the

*School Science and Mathematics, October, 1930, p. 757.

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actual from the estimated populations in per cent are as follows:

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The effect of each of these variations, with the exception of New York, is to place farther to the west the center of population, while those of Texas and California would tend toward a southern migration of that center.

The actual figures for California throw further light on the matter. The Census for 1930 gives this state 5,672,009 inhabitants. The estimated number (as of Jan. 1, 1930) used in the prediction was 4,730,000, this number having been determined by extrapolation from the yearly estimates of the Department of Commerce for the years 1920-1928 inclusive. It can readily be seen that an increase of 942,000 in the population of California alone results in a shift of the center of population westward by as much as 13 miles.

The uncertainty of these estimates of the Department of Commerce has suggested a comparison of similar estimates made a decade earlier with the census figures of 1920. Such a comparison shows variations of the same character as have been noted for 1930. In fact, for three of the states, the estimates differed by more than 20% from the actual populations as given by the census, while in six others the difference exceeded 10%. Nevada had a population of 81,875 in 1910, and the estimates indicated a steady increase to 115,000 in 1919. The census of 1920 gave it 77,407.

Perhaps the most significant outcome of this discussion is the manner in which it reveals the unreliability of the yearly population estimates. One is tempted to feel that there is, at least in some cases, insufficient basis for the making of such estimates and that they are worth little more than a guess.

EASTERN ASSOCIATION OF PHYSICS TEACHERS*

REPORT OF COMMITTEE ON NEW APPARATUS

By John C. Packard, *Chairman, High School, Brookline, Mass.*

Mr. Packard showed in operation a Junker calorimeter so arranged as to deposit, in a beaker, the condensed water derived from the flame of a Bunsen burner. This shows, in a somewhat dramatic manner, that water may be actually formed on a fairly large scale —200 c.c. per hour in this case—by the combustion of hydrogen in the air. Quite an improvement over the familiar mist-on-the-chimney experiment.

Mr. Gibbons of the Brighton High School presented an Exercise upon the Testing of a Commercial Jackscrew as typical of his so-called "summary exercises." This involves in one experiment a complete review of the fundamental principles underlying the chapter on Mechanics.

A representative of the Central Scientific Company showed their well-known form of apparatus "A"—boiler used in experiments on testing a mercury thermometer, etc.—made of Pyrex glass, with an electric coil inserted to take the place of the gas jet in heating the water. There was also shown a semi-automatic wet-and-dry bulb

*Additional items in the report of 119th meeting, which was published in the February issue.

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hygrometer, compact and beautifully made, that would make an attractive addition to any laboratory or lecture room.

Miss Payne presented at some length a very interesting and timely address on "Analyzing the Universe." She dwelt particularly on the various types of systems of nebulae, pointing out that there are many other solar systems making galaxies as shown by the spiral nebulae. She also spoke of the importance of spectroscopic analysis to the astronomer. Many lantern slides were shown illustrating the material presented in the address.

W. W. OBEAR, *Secretary.*

TEACHING FLUID PRESSURE

By WM. A. PORTER, *Chisholm, Minn.*

In teaching the subject of water or fluid pressure I have found the following "short cut" useful. After developing the topic in the usual manner and performing the experiment proving that the pressure is equal to the height times the density of the liquid, the formula P=HD is placed on the board. Since most of our practical problems use the English units it is necessary to evaluate the pressure in pounds per square inch. But P=HD gives us the pressure in pounds per square foot. Therefore it is necessary to divide by 144 to obtain the pressure in pounds per square inch.

$$\text{Then, } P = \frac{HD}{144}.$$

But D is usually constant at 62.4 lbs. per cubic foot.

$$\text{Therefore } P = \frac{H \times 62.4}{144}$$

After solving a few type problems using this formula it is easily shown that P and H may vary but that D and 144 do not. Therefore the equation may be written $P = H \times \frac{62.4}{144}$.

Since the fractional part of this equation is clumsy we can reduce it to decimals

$\frac{62.4}{144}$ equals .433, so our equation now reads $P = H \times (.433)$

A column of water one foot high will have a pressure of:

$P = 1 \times (.433)$

or P=.433 lbs. per square inch, the pressure produced by a column of water 1 foot high.

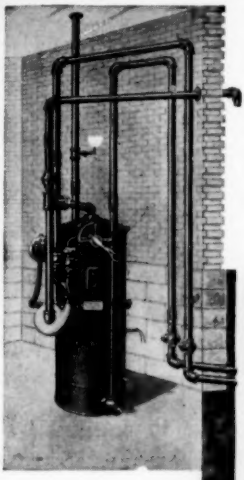
This factor may be used in a variety of ways, such as: Find the head of water necessary to produce a pressure of 60 lbs. per square inch.

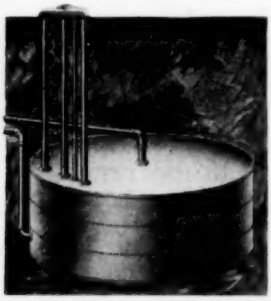
Formula $P = H \times (.433)$ But P is given as 60 and H is the head, or height in feet to be found.

Substituting these values in the formula we have

$$60 = (.433) \times H \quad \text{or} \quad H = \frac{60}{(.433)}$$

H=138 ft. of water necessary to produce a pressure of 60 lbs. per square inch.





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In sea water it is necessary to change the formula to correspond with the change in density of the water. Average density of sea water being 1.025 times that of pure water.
For Sea Water:

$$P = \frac{H \times 62.4 \times 1.025}{144}$$

$$P = \frac{H \times 64}{144}$$

$$P = H \times \frac{64}{144}$$

$$P = H \times (.44)$$

For example:
A diver descends to a depth of 250 feet in sea water having a specific gravity of 1.025. What pressure in pounds per square inch is exerted by the water on the diver?

Formula for sea water:
$P = H \times (.44)$ Given H equals 250 feet
To find P
Substituting these values in the above equation
$P = 250 \times (.44)$
P = 110 lbs. per square inch pressure on the diver.

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